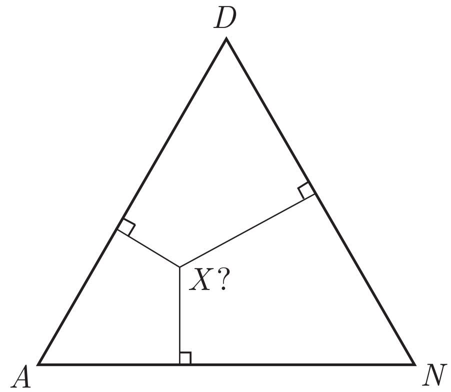
3 *The Ladybug PiCMIc*

PROBLEM

Equilateral triangle DAN has point X inside it.



Where should point X be placed so that the three *perpendicular* segments to the three sides of the triangle have a minimum total length?

So, the first task is to sketch a Dorito, sometimes called an "equilateral triangle"...

In Sketchpad, you can draw a perpendicular line... then what? I swear I once saw this picture on the outside of an Arby's in Hollywood.

Important Stuff.

1. Two numbers add up to 200, and their product is 9,919. Find the two numbers.

Look! Over there! Numbers!

2. (a) Two numbers add up to 30, and their product is 161. Find the two numbers.

(b) Two numbers add up to 30, and their product is $225 - n$. Find the two numbers.

(c) Two numbers add up to 30, and their product is 289. Find the two numbers.

Say, isn't 289 a perfect square? 8-15-17, or something like that? Oh, never mind.

3. For each point, determine whether or not it is 10 units away from $(13, -7)$.

(a) $(5, 3)$ (c) $(10, -16)$
(b) $(7, 1)$ (d) (x, y)

4. For any rectangle you can assign a point (l, w) in a coordinate plane, defined by the length and width of the rectangle. Rectangle $RAUL$ has length 10 and width 15.

Hope you brought some rectangular graham crackers to the PiCMIc.

(a) Plot four points that all correspond to rectangles similar to $RAUL$.
(b) How many rectangles are similar to $RAUL$? Plot them all.
(c) Find a rectangle similar to $RAUL$, but whose perimeter in units is larger than its area in square units.
(d) Find a rectangle similar to $RAUL$ whose perimeter and area have the same numerical value.

Two shapes are *similar* if one is a scaled copy of the other, like a napkin and a Cheez-It.

5. Darryl is so one-dimensional, he lives on the x-axis. He needs to travel once each to $(1, 0)$, $(3, 0)$, and $(17, 0)$. Where should Darryl live to minimize his total travel distance? You have 30 seconds to guess... go!

How one-dimensional is he? His width and height are infinitesimal! *Ha!*

6. (a) Point Q has coordinates $(3, 0)$ and point P has coordinates $(x, 0)$. Define $f(P)$ to be the distance from point P to point Q. What does the graph of $f(P)$ look like?
(b) Point R has coordinates $(17, 0)$. Now define $f(P)$ as the sum of the distances PQ and PR. What does the graph of $f(P)$ look like?
(c) Point S has coordinates $(1, 0)$. You take it from here.
(d) Where should Darryl live to minimize his total travel distance?

OMG BBQ

7. Find three rectangular boxes where the numeric value of their surface area equals the numeric value of their volume.

One of them is a common shape of Jell-O.

8. Bill bought two briefcases this weekend. Remarkably, both have the same surface area and both have the same volume. And yet, the two briefcases are not the same size. Find some possible dimensions for Bill's briefcases.

However, one of the briefcases was 80% off. Ask Bill all about it. He'll be thrilled to tell you all the details.

9. Find two triangles where the numeric value of their perimeter equals the numeric value of their area.

Or not. There may not even be two such triangles. We ain't tellin.

Neat Stuff.

10. Find three pairs of positive numbers a and b, with $a \geq b$, that satisfy

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{2}$$

11. Find all rectangles with integer side lengths whose area, in square units, is exactly double the perimeter in units.

12. Go back and do problems 16 through 18 from Day 2 if you haven't already. They are *cool*.

Problem 12 is the ladybugs' favorite.

13. Let $p(x) = x^4 - 2x^3 + 3$ and $q(x)$ is the *remainder* when $p(x)$ is divided by $(x-2)(x-5)\ldots$ in other words, $x^2 - 7x + 10$.

But Problem 13 is like ants at the PiCMIc.

(a) Complete this table of values for $p(x)$:

x	$p(x)$
0	
1	
2	
3	
4	
5	

(b) Complete this table of values for $q(x)$:

x	$q(x)$
0	
1	
2	
3	
4	
5	

Notice anything interesting?

14. Let $f(x) = 3x^2 - 10x + 21$. Use polynomial long division to find a linear function that agrees with $f(x)$ when $x = 3$ and when $x = -5$.

15. Let $f(x) = 3x^2 - 10x + 21$. Ooh, the same function.

(a) Use division to find $g(x)$, a linear function that agrees with $f(x)$ when $x = 3$ and when $x = 3$.

This isn't a typo. Use the method from Problem 14. *THIS ISN'T A TYPO!*

(b) Graph $f(x)$ and $g(x)$ on the same axes, using Sketchpad or a graphing calculator. What do you notice?

16. Find the equation of the tangent line to $f(x) = x^3$ at $x = 2$. Do *not* use the calculus!

17. Do the problem in the box, but this time use a generic triangle instead of an equilateral one. What happens?

Okay, who brought the generic-brand Doritos to the PiCMIc...

18. (a) Point A has coordinates $(0, 0)$ and point P has coordinates (x, y). Define $f(P)$ to be the distance PA. What does the graph of $f(P)$ look like?

(b) Point L has coordinates $(8, 0)$. Now define $f(P)$ as the sum of the distances PA and PL. What does the graph of $f(P)$ look like?

(c) Point M has coordinates $(3, 6)$. You take it from here.

19. In the box from Session 2, Art runs just as fast to the river as he does away from it. But in reality, Art runs twice as fast when he's thirsty as he does when he's not. How does this change the sketch and its solution? Can you model this in Sketchpad? On a graphing calculator? What is Snell's Law?

20. What is the largest prime number ever used in the lyrics of a Top 40 song?

Tough Stuff.

21. Find the equation of a parabola that is tangent to the function $f(x) = x^4 - 2x^3 + 3$ at $x = 1$, and also intersects $f(x)$ at $x = -1$. Under no circumstances are you to use the calculus for this problem! We'll know!

22. Given a positive integer k, there are a number of values of b so that the quadratic $x^2 + bx + kb$ is factorable over the integers. Determine, based on k, how many such values of b there are.

23. The quadratic equation $x^2 - 10x + 22 = 0$ has two roots.

(a) Find a quadratic whose roots are the *squares* of the roots of $x^2 - 10x + 22 = 0$.

(b) Find a quadratic whose roots are the nth powers of the roots of $x^2 - 10x + 22 = 0$.

24. Find all integer solutions to this system of equations:

Nope, there's more.

$$a + b = cd$$
$$c + d = ab$$

25. There's a point inside most triangles that forms three 120° angles with segments to the three vertices. A *Matsuura triangle* is a triangle whose side lengths are all integers, *and* whose three interior segment lengths from the 120° point are also integers. Find some Matsuura triangles, or prove they do not exist.

We'll keep these on the problem sets until someone gets 'em! Just kidding. Or are we. . .