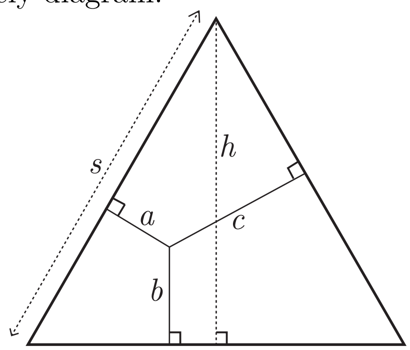
4 *uPCoMIng attractions*

Important Stuff.

1. Look at this lovely diagram!

Like Roxette, it's got the look.



(a) Find an expression for the area of the equilateral triangle pictured above.
(b) Find a triangle with area $\frac{1}{2}as$. It might be hiding!
(c) Find a second expression for the area of the equilateral triangle, then work your magic.

It's okay for the expression to have more than one variable, just like it's okay not to have the box come first.

Poof goes the proof, and boom goes the dynamite.

PROBLEM

Sketch a right triangle that stays a right triangle when you move its points around. (What would you need to construct first?)

Then, build three equilateral triangles on the outside of the right triangle by using each side of the right triangle as the base for an equilateral triangle.

Find a relationship between the areas of the equilateral triangles.

We interrupt today's Important Stuff with this friendly and informative moment: the Pythagoreans were a bunch of squares.

PROBLEM

Redo the above construction using a scalene triangle as the starter, building equilateral triangles along each side.

Use this sketch to *construct* the 120° point. See if you can figure out how, and why. If you've already built this construction, try to find another. The sketch from Day 3 might help!

2. Now that you are in new groups, ask your tablemates for the dimensions of some rectangles whose area and perimeter have the same numeric value. Compile a list. For each rectangle, calculate

$$\frac{1}{L} + \frac{1}{W}$$

where L and W are the length and width of each rectangle.

Gotta make my mind up. . . which seat can I take?

3. Rectangle $ARON$ has length 20 and width 15.

(a) In a coordinate plane, plot (l, R), where l is the length of $ARON$ and R is the ratio of the area of $ARON$ to the perimeter of $ARON$.

(b) Find several rectangles similar to $ARON$, and plot them on the same axes.

(c) Find the one rectangle similar to $ARON$ whose perimeter and area have the same numeric value.

4. Tom starts with a 3 by 4 by 5 box.

(a) Mimi finds a box similar to Tom's box whose surface area and volume have the same numeric value. What are the dimensions of Mimi's box?

(b) Call the dimensions of Mimi's box T, O, and M. Find the exact value of

$$\frac{1}{T} + \frac{1}{O} + \frac{1}{M}$$

What!!

$\frac{1}{O}$ is not quite the same as dividing by zero. . .

5. For each point, determine whether or not it is twice as far from $(20, 0)$ as it is from $(5, 0)$.

This one isn't multiple choice, either!

(a) $(10, 0)$

(b) $(-6, 8)$

(c) $(8, 6)$

(d) (x, y)

6. Two positive numbers multiply to 49. What is the largest and smallest sum possible?

7. Which of the following is the worst possible title we could have used this week?

Write in part (j) and submit your own!

(a) hyPoglyCeMIa

(b) PsyChosoMatIc

(c) y-IMterCePt

(d) I, MuttonChoPs

(e) nIMCompooP

(f) I, creaMCuPs

(g) rePubliCaMIze

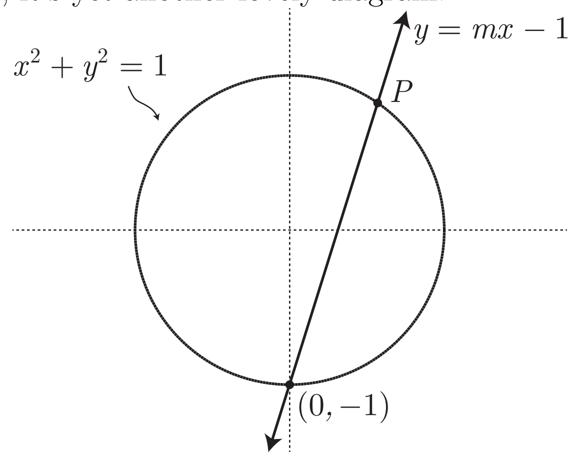
(h) I, MicroChiP

(i) PoliCyMakIng

(j)

Neat Stuff.

8. Look, it's yet another lovely diagram!



And it goes, "Na na na na na, na na na na na, na na NA na na na..." It's got the look. Sorry.

For each value of m given, determine the exact coordinates of point P.

(a) 2
(b) $\frac{3}{2}$
(c) $\frac{4}{3}$
(d) 4
(e) 10
(f) $\frac{a}{b}$

9. Let $f(x) = x^3 - 6x^2 + 4x + 8$. For each quadratic below, find the remainder when $f(x)$ is divided by the quadratic, then plot $f(x)$ and the remainder on the same axes (using techmology). What do you notice?

(a) $(x - 2)(x - 6) = x^2 - 8x + 12$
(b) $(x - 2)(x - 5) = x^2 - 7x + 10$
(c) $(x - 2)(x - 4) = x^2 - 6x + 8$
(d) $(x - 2)(x - 3) = x^2 - 5x + 6$

Sketchpad can do this too? Indeed.

10. Let $f(x) = x^3 - 6x^2 + 4x + 8$. Do what you did on the last problem. What do you notice?

(a) $(x - 2)^2$
(b) $(x - 3)^2$
(c) $(x - 4)^2$
(d) $(x - 5)^2$

11. Find the equation of the tangent line to $f(x) = x^4$ at $x = 1$.

12. Mary draws an equiangular octagon with alternating sides of length 3 and length 2. She then challenges you to find the point inside the octagon with the minimum total of the *eight* altitudes drawn from that point to the sides of the octagon. Go!

All the angles in this octagon are 136 degrees within ± 1 margin of error. As with triangles, some of the altitudes may extend outside the shape.

13. If you draw the altitudes from any interior point to all sides of an equilateral triangle, the altitudes' lengths add up to a constant. What about these shapes? For those that work, prove it; for those that don't, explain why they don't work.

(a) a square
(b) a rectangle
(c) a rhombus
(d) a parallelogram
(e) a regular pentagon
(f) an equilateral hexagon (not regular)
(g) an equiangular hexagon (not regular)

14. The sum of two numbers is s and the product is p. Find the sum of the...

(a) squares of the two numbers.
(b) cubes of the two numbers.
(c) fourth powers of the two numbers.
(d) ...a generalization?

I got this, you got this... Now you know it!

Tough Stuff.

15. Take a triangle, and move its points according to the rule $(x, y) \mapsto (ax + by, cx + dy)$. Find integer values for $a, b, c,$ and d so that the new shape has a smaller area than the original shape, but still *some* area.

16. So we've discovered that this 120° point gives the least possible total distance to the three vertices. But what about other points? They're worse, but some are not much worse. Indeed, the shape of the points that are *equally bad* is interesting. What's it look like? What's it look like if you move outside the original triangle?

17. Find several triangles that have integer side lengths (with no common factors) and a 120° angle. Generalize?

18. Find some Matsuura triangles, or prove they do not exist. (See previous sessions for the definition.)

19. Find this sum exactly:

$$0 + \frac{1}{100} + \frac{4}{10000} + \frac{9}{1000000} + \cdots + \frac{n^2}{10^{2n}} + \cdots$$