5 *Week 2: The Slurpe-NING*

Hey, welcome back to the class. We know you'll continue to learn a lot of mathematics here—some new tricks, some new perspectives on things you might already know about. A few things to recall about how the class is organized:

- **Don't worry about answering all the questions.** If you're answering every question, we haven't written the problem sets correctly.
- **Don't worry about getting to a certain problem number.** Some participants have been known to spend the entire session working on one problem (and perhaps a few of its extensions or consequences).
- **Stop and smell the roses.** Getting the correct answer to a question is not a be-all and end-all in this course. How does the question relate to others you've encountered? How did others at your table think about this question?
- **Respect everyone's views.** Remember that you have something to learn from everyone else. Remember that everyone works at a different pace.
- **Learn from others.** Give everyone the chance to discover, and look to those around you for new perspectives. Resist the urge to tell others the answers if they aren't ready to hear them yet. If you think it's a good time to teach everyone semiperimeter formulas, think again: problems should lead to the appropriate mathematics rather than requiring it. The same goes for technology: the problems should lead to appropriate uses of technology rather than requiring it. Try to avoid using technology to solve a problem "by itself". There is probably another, more interesting, way.
- **Each day has its Stuff.** There are problem categories: Important Stuff, Neat Stuff, Tough Stuff, and maybe more. Check out Important Stuff first. The mathematics that is central to the course can be found and developed in Important Stuff. After all, it's Important Stuff. Everything else is just neat or tough. If you didn't get through the Important Stuff, we noticed... and that question will be seen again soon. Each problem set is based on what happened before it, in problems or discussions.

Today is FREE SLURPEE DAY! Seriously.

Important Stuff.

1. Two positive numbers multiply to 81. What is the largest and smallest sum possible?

PROBLEM

Start a new Sketchpad sketch and follow these steps.

- Draw a circle.
- Draw a line that intersects the circle using two points that are outside the circle, and on opposite sides. Label one of these outside points A.
- Select the line and the circle, then click on "Construct" and "Intersection."
- Label the constructed points P and Q.

1. Find the location of Q that maximizes the sum $AP + AQ$. You can drag the other unlabeled point on the line to move P and Q. ♡
2. Find the location of Q that maximizes the product $AP \cdot AQ$.
3. Find the location of Q that minimizes the sum $AP + AQ$.
4. Move point A so that it is inside the circle. For this new location of A, find the location of Q that maximizes the product $AP \cdot AQ$.

Say, this box is nearly a square. Circle gets the square! Paul Lynde would be proud, or at least snickering.

Two, two, two intersections in one!

We ♡ math! We ♠ our dog! We ♣ all weekend! We ♢ a shrubbery!

2. Find a rectangle similar to a 7 by 11 rectangle and has the same numerical value for area and perimeter.

3. Todd starts with a 5 by 6 by 20 box.

(a) Betul finds a box similar to Todd's box whose surface area and volume have the same numeric value. What are the dimensions of Betul's box?

(b) Call the dimensions of Betul's box T, O, and D. Find the exact value of

$$\frac{1}{T} + \frac{1}{O} + \frac{1}{D}$$

We were *this* close to putting a 7 by 11 by 61.6 box on the problem set instead. Oops, we just did!

4. For of each these points, determine whether or not it is three times as far from $(9, 0)$ as it is from $(1, 0)$.

(a) $(0, 3)$
(b) $(7, 0)$
(c) $(-2, 2)$
(d) $(-3, 0)$
(e) $(1, -5)$
(f) (x, y)

5. Do the "*SAM*" problem (number 7) from Day 2 if you haven't already.

6. Let $s = 2 + i$ and $m = 4 + 6i$ be complex numbers. Find each of these.

(a) $s + m$
(b) $s - m$
(c) $3s + 3m$
(d) sm (the product)
(e) is
(f) im

When you see i^2, make it -1. The number i is the imaginary square root of -1.

What is it? That depends on the definition of is.

7. Simplify these seemingly nasty-looking expressions that involve square roots of square roots of negative numbers.

(a) $\sqrt{(3 + 4i)(3 - 4i)}$
(b) $\sqrt{(5 - 12i)(5 + 12i)}$
(c) $\sqrt{(15 + 8i)(15 - 8i)}$
(d) $\sqrt{(x + yi)(x - yi)}$

8. How far is each of these points from the origin?

(a) $(3, 4)$
(b) $(5, -12)$
(c) $(15, 8)$
(d) (x, y)

The coordinates of the origin are hole numbers.

9. (a) Draw a graph of all the points (x, y) that are 5 units from the origin.
(b) Write an equation for the graph you just drew.

10. Find all 12 complex numbers $a + bi$ with integers a, b so that

$$\sqrt{(a + bi)(a - bi)} = 5$$

A complex number with integers a, b is called a *Gaussian integer*. You can look up one of the 12 answers.

Useless Stuff.

11. What is $6 \div 2(1 + 2)$?

12. Solve for X:

$$(X^{10} + (\text{mushroom})^2)^4 + X^{10} = \text{snake}$$

"In the Middle Ages, people in convents were not allowed to eat beans because they believed something about them we now know isn't true. What?" Paul Lynde: "Well, I know they took a vow of silence..."

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Neat Stuff.

13. For each point P, determine if it is twice as far from the *line* $x = 2$ as it is from the point $(-1, 3)$.

(a) $(5, 3)$
(b) $(0, 3)$
(c) $(-2, 4)$
(d) $(-3, 2)$
(e) $(-2, b)$
(f) (x, y)

Is $x = 2$ vertical or horizontal? Decide by asking what points are on its graph.

14. Square $ZACK$ has one vertex at $A(1, 0)$. The center of the square is the origin $O(0, 0)$. Find the coordinates of the other vertices.

15. Find all four solutions to the equation $x^4 - 1 = 0$.

Here's a useless reminder that $a^2 - 9 = (a+3)(a-3)$. Or *is* it?

16. Equilateral triangle DEB has one vertex at $B(1, 0)$. The center of the triangle is the origin $O(0, 0)$. Find the coordinates of the other vertices.

17. Find all three solutions to the equation $x^3 - 1 = 0$.

You can factor something by using long division. Divide, and if the remainder is zero, woot.

18. Regular hexagon $BUSHRA$ has one vertex at $A(1, 0)$. The center of the hexagon is the origin $O(0, 0)$. Find the coordinates of the other vertices.

19. Find all six solutions to the equation $x^6 - 1 = 0$.

20. Let $z = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$. Take powers of z until it becomes awesome, then tell us the value of z^{101}.

The value of z^{101} is contributing awesome rock to Niagara Falls and the surrounding area!

21. Find a complex number z so that $z^{12} = 1$ but no smaller positive integer n has $z^n = 1$. Did we say "a"? We meant "all" of them.

More Useless Stuff.

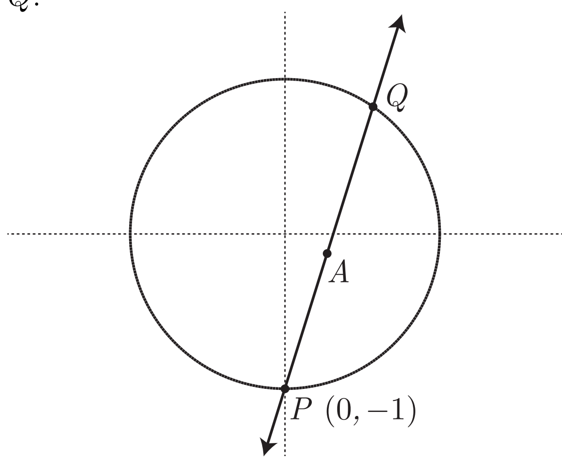
22. A free Slurpee is a truncated cone with a volume of 7.11 fluid ounces. Determine its total surface area.

23. Seriously, go get a free Slurpee!

"In one state, you can deduct \$5 from a traffic ticket if you show the officer...what?" Paul Lynde: "A ten dollar bill."

Neat Stuff.

24. Let P be a point at $(0,-1)$ and A be a point inside the unit circle. Let the line AP intersect the unit circle at point Q.



Calculate $f(A) = AP \cdot AQ$ for each of these points by determining the lengths AP and AQ.

(a) $A = (\frac{1}{2}, 0)$
(b) $A = (\frac{1}{4}, 0)$
(c) $A = (-\frac{1}{2}, 0)$
(d) $A = (\frac{1}{3}, -\frac{1}{2})$
(e) $A = (\frac{1}{3}, \frac{1}{2})$
(f) $A = (x, y)$

What happens if A is outside the circle?

"Why do sheep sleep huddled up?" Paul Lynde: "Because Little Boy Blue's a weirdo!"

Tough Stuff.

25. Suppose n is a positive integer. Find a rule that determines whether or not there is a right triangle with area n and rational numbers as side lengths.

26. Complete this long division problem, where all the missing digits are marked with an X. (There is no remainder.)

$$\begin{array}{cccccccccccc}
&&&&&&&X&X&8&X&X \\ \cline{5-12}
X&X&X&\big)&X&X&X&X&X&X&X&X \\
&&&&&X&X&X&&&& \\ \cline{5-8}
&&&&&&X&X&X&X&& \\
&&&&&&&X&X&X&& \\ \cline{7-10}
&&&&&&&&X&X&X&X \\
&&&&&&&&X&X&X&X \\ \cline{9-12}
\end{array}$$

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PCMI rocks! Woot woot!