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7 *Boom Win-NING*

PROBLEM

Pour some salt on some triangles... Umm... Yeah, we'll explain what to do.

Discuss the answers to these questions with your group. Feel free to use this GSP file to investigate:

`http://www.tinyurl.com/saltgsp`

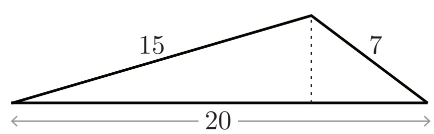
- What is so special about where the top of the salt heap is located?
- What is so special about where the ridges are located?
- Why is the top of the salt heap the same ________ to the three ______?

Today's problem in the box is a real happe-NING brought to us by our very own Troy Jones. Pour some salt on me! One more, in the name of math.

Charles Nelson Reilly would have the definitive answer here.

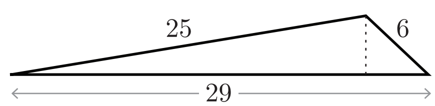
Important Stuff.

1. Find the area of this triangle.



This could be called a Boxx problem, or a Rapinoe problem, but most people will probably call it a Wambach problem.

2. Find the area of this triangle.



PROBLEM

Snag this file `http://www.tinyurl.com/circlegsp` and adjust the circle until it is the largest one that doesn't pass outside the triangle. What is the radius of this circle?

3. (a) In your diagram from the second box, find a triangle with area 20. It might be hiding!

(b) Find an expression for the area of the entire triangle in terms of things that aren't the height of the triangle.

A look back at problem Solo from day Sauerbrunn might be helpful.

4. Consider the complex numbers $s = 2 + i, a = 4 + i, m = 4 + 6i$.

(a) Plot and label s, a, and m in a complex plane.

(b) Multiply each number by i, then plot and label each of the new numbers in the *same* complex plane.

(c) Multiply each of s, a, and m by i *twice*, then plot and label each of the new numbers in the same plane.

(d) Three times? Four times? Five times? Thirteen times? 101 times?

Plot the complex number $x + yi$ at the point (x, y) in the *complex plane*. Don't worry, there are no snakes.

This all seems so familiar somehow. . .

5. Let $z = 1 + i$. Plot each of these in the same complex plane, and find the magnitude of each.

(a) z

(b) z^2

(c) z^3

(d) z^4

(e) z^5

6. Craig stands at the origin $(0, 0)$ and stares at the powers of $1 + i$ as they are built. Describe what happens to the powers from his perspective: where do they go? how far away?

7. Let $z = 3+2i$. Find the *magnitude* of each of these complex numbers.

(a) z

(b) z^2

(c) z^3

(d) z^4

z = Rampone + M$i$tts. But z should really just pick a value and stick to it. Y U NO STOP CHANGING Z!

8. Let $z = \frac{3}{5} + \frac{4}{5}i$. Plot and label each of these on the same complex plane. Approximations are fine here.

(a) z

(b) z^2

(c) z^3

(d) z^4

(e) z^5

9. Mahen stands at the origin $(0, 0)$ and stares at the powers of $\frac{3}{5} + \frac{4}{5}i$ as they are built. Describe what happens as accurately as you can. How is it similar to what happens with the powers of $1 + i$? How is it different?

Fortunately, Mahen is not a Care Bear, otherwise we'd all be in trouble.

Neat Stuff.

10. If $a = 3 + 4i$ and $b = 5 + 12i$, find the magnitude of a, the magnitude of b, the magnitude of ab.

11. (a) Find the magnitude of $w = 2 + i$.
(b) Perform an operation to w that results in a complex number that has magnitude 5.

Yes, we mean 5, not $\sqrt{5}$.

12. Do stuff to take each of these complex numbers and produce a primitive Pythagorean triple.
(a) $4 + i$
(b) $8 + 3i$
(c) $15 + 4i$
(d) $16 + 7i$
(e) $23 + 2i$
(f) $42 + 9i$

Triples are fun! This is Hurley's favorite problem. Hurley was so 2008. Boom. Boom. Pow. It's okay, we don't get it either.

13. (a) Three numbers add up to 14 and multiply to 72. Find both sets of three positive integers that work here.
(b) Use the results to find two boxes whose surface area and volume equal one another.

That's three numbers, not two and a half numbers. Boom.

14. Can two non-congruent triangles have the same perimeter and area? Explain!!

15. A triangle has side lengths a, b, and c. Without using fancypants formulas, find its height. What, there's more than one height? Fine, the area then.

Guys like me from Wall Street wear fancypants. Boom.

16. The magnitude of $3 + 4i$ is 5. Find *all* the Pythagorean triples with hypotenuse 125 (including non-primitive ones).

17. Use the concepts from problem 10 to find a primitive Pythagorean triple with hypotenuse 1105. Then another one! How many *are* there??

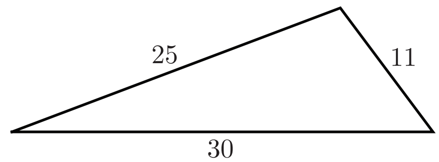
What's so special about 1105 anyway?

Tough Stuff.

18. (a) Find a way to generate all of the Pythagorean triples in which *the two leg lengths* are one away from each other. One example is 21, 20, 29.
(deux) Find a way to generate Pythagorean triples that are incredibly close to being 30-60-90 right triangles. They can't be, but they can be super close!

These problems are for the Hot Shots...

19. Besides being Heronian, this triangle has some other interesting feature. Find a way to generate more that have its special property.



20. Where are our Superheronian triangles! Bah!

21. $\frac{A}{BC} + \frac{D}{EF} + \frac{G}{HI} = 1$. Here, each letter is a unique and distinct number from 1 to 9, and the denominators are two-digit numbers. Find a solution!

Wait, are you reading this before doing the problems at the beginning? Get back to the first page, *now*!

22. As you keep taking powers of $z = \frac{3}{5} + \frac{4}{5}i$, will they eventually wrap around onto themselves? In other words, are there powers k and m with $z^k = z^m$ for this z?

23. In triangle ABC, angle A is twice angle B, angle C is obtuse, and the three side lengths a, b, and c are integers. Determine, with proof, the minimum possible perimeter.