Do not cite or quote, or we will cast Expelliarmus on you.

9 *Stupefy-NING*

PROBLEM

Download this file for today's sketch:

`http://tinyurl.com/complexgsp`

Let v be a complex number with magnitude 2.

- Draw a shape to indicate where v could lie in the complex plane.
- Pick a value of v and square it: where does it go? Describe all the possible places where v^2 could lie.

Accio Sketch!

Magnitude 2, like Fred and George Weasley? Oh.

Sadly the Sorting Hat is on holiday. He'll be back Monday.

Important Stuff.

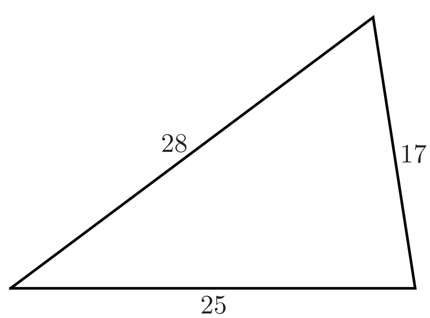
1. Use complex numbers to make three ridiculous primitive Pythagorean triples.

Riddikulus!

2. Find the area of this triangle. Keep careful track of your steps and do not use any "instant winner" formulas.

Expecto Heronus!

While it is possible to "eyeball" the triangle by picking just the right altitude, please additionally work through the calculations. They'll be helpful on the next page.



3. Explain why the area of a triangle is given by

$$A = \frac{1}{2}Pr$$

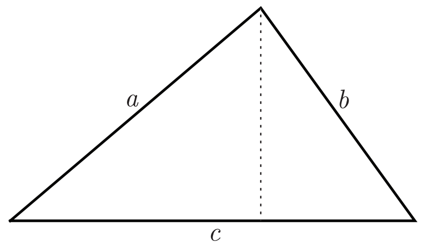
where P is the triangle's perimeter and r is the radius of its incircle.

4. Find the incircle radius of the triangle up there.

Wingardium Leviosa!

5. Find the area of this triangle by following the same steps you followed in problem 2.

Y U NO GIVE ME SIDE LENGTHS!!!!!



6. Find the incircle radius of a triangle whose side lengths are a, b, and c.

Specialis Revelio!

7. Take an 8-15-17 right triangle and a 5-12-13 right triangle and slap them together in some way to form a triangle with integer side lengths and integer area. You may need to scale one or both triangles first.

Triangles with integer side lengths and integer area are called *Hermionian triangles*.

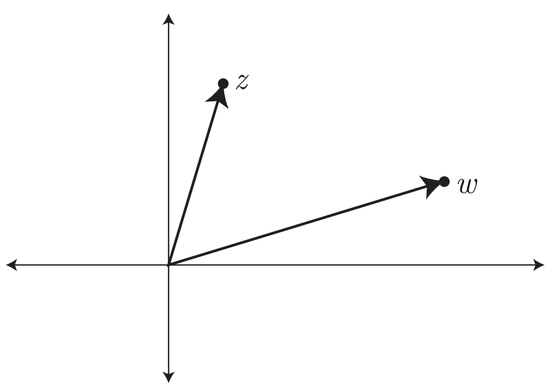
Engorgio!

8. Find at least two more ways to turn the 8-15-17 and the 5-12-13 into Hermionian triangles. Sweet, huh?

Ten points to Gryffindor!

9. Use your ridiculous Pythagorean triples to make a few ridiculous Hermionian triangles. Woo hoo!

10. The complex numbers z and w are plotted below. Plot $z + w$ and indicate how you know where it should be.



Neat Stuff.

11. Compute $(1 + 2i)(1 - 2i)$. Here, $1 - 2i$ is the conjugate of $1 + 2i$.

12. Find the magnitude and direction of $1 + 2i$.

Sketchpad can help you measure angles; remember, the "direction" angle is measured counterclockwise from the positive real axis.

13. Here's three complex numbers: $j = 3+i, m = 3+2i, i = i$. They can be connected to make a triangle.
(a) Plot these three numbers in the complex plane.
(b) Multiply each number by $1+2i$, then plot the results in the complex plane.
(c) Describe, as precisely as possible, how the new points' locations compare to the old.
(d) How does the area of the new triangle relate to the area of the old triangle?

Prior Incantato!

14. Find the surface area and volume of a 4 by 10 by 15 box and a 5 by 6 by 20 box.

15. Multiply these out until you don't feel like it anymore.
(a) $(x-4)(x-10)(x-15)$
(b) $(x-5)(x-6)(x-20)$
(c) $(4x-1)(10x-1)(15x-1)$
(d) $(x-\frac{1}{4})(x-\frac{1}{10})(x-\frac{1}{15})$

Impedimenta!

16. Find the dimensions of a box with the same total edge length and volume as the box with dimensions $\frac{1}{4}$ by $\frac{1}{10}$ by $\frac{1}{15}$.

17. So now you've got a cool formula for the area of a triangle given its sides. But those crazy formula people were busting out this crazy formula:

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

Can you turn your cool formula into this one with an s?

This thing s is called the *semiperimeter*.

18. Yesterday, Mary showed a sketch (available on the PCMI @ Mathforum website) where a circle of radius 2 is built, then a triangle is built using three points of tangency.
(a) Why was this so cool?
(b) Investigate: is it possible to construct triangles with specific characteristics using this concept?

The file is called "mary's-cool-triangle.gsp". Blame Darryl! But it is cool.

19. Find a different Heronian triangle with the same perimeter and area as the 17-25-28 triangle.

Geminio!

20. A triangle has side lengths 15, 7, and x, and its area and perimeter have the same numerical value. Find all possible values of x.

21. Find an equation for all the points that are α times as far from $(-1, 4)$ as from the line $y = 1$. (Let α be a positive number.) Describe how the shape created depends on the value of α.

Make sure what you find agrees with what you learned through Problems 12–14 from Day 8.

22. Suppose z is a complex number with magnitude 1 and direction θ. Then $z = a + bi$ with $a = \cos\theta$ and $b = \sin\theta$.
(a) Calculate z^2 directly by squaring $z = a + bi$. Wow!!
(b) Find a formula for $\cos 3\theta$.

If you're not working with trigonometry regularly, these last four will be relatively boring problems and are easily skipped.

23. Suppose z and w have magnitude 1, z has direction α and w has direction β. Let $z = a + bi$ and $w = c + di$.
(a) Calculate zw.
(b) What are the magnitude and direction of zw?
(c) What's the formula for $\sin(\alpha + \beta)$?

24. If $\tan A = a$ and $\tan B = b$, find a *simple* way to compute $\tan(A + B)$ without formulas.

Multiply! Find the right complex numbers and you're off to the races.

25. Pick angles A, B, and C that form a triangle. (You know what we mean.) Now let $\tan A = a, \tan B = b, \tan C = c$. Which is bigger, the product abc or the sum $a+b+c$? Try another triangle and compare. WHAT!

Petrificus Totalus!

Useless Stuff.

26. Solve for X.

Check my time, it's X.

$$\begin{aligned} Y_{est} &= X - 1 \\ 2D &= X \\ 2M_0 &= X + 1 \\ X + 2 &= \text{afterwards} \end{aligned}$$

Tough Stuff.

27. A triangle is uniquely determined by its side lengths, so it makes sense there is a formula for the area in terms of the three side lengths. Did you know that a triangle is also uniquely determined by its median lengths? Uh-huh! That means there is a formula for the area in terms of the lengths of the three medians. Go find it and rejoice because it's awesome.

Confundo!

28. There is also a formula for the area of a triangle in terms of the lengths of its three altitudes. It is less awesome but still possible.

Crucio!

29. There is also a formula for the area in terms of the lengths of its three angle bisectors. Good luck with that.

30. Prove that every Pythagorean triple must have a multiple of 3, a multiple of 4, and a multiple of 5 in it somewhere.

Looks like we've come to the end of the road. Still, I can't let go. I want a perfect body of problems. You may hate me but it ain't no lie. Next week it will be a whole new world... don't you dare close your eyes! It's 3 am, I must be lonely.