Million Dollar Mathematics of Game Shows

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PRIZES!

Want to win?

We'll need some volunteers for games.

You may leave here with *fabulous prizes*!

(Disclaimer: prizes are very unlikely to be "fabulous".)

PRIZES!

Speaking of which…

Sarah Trask, *come on down!*

You're our first contestant!

Deal or No Deal

$0.01	$1
$0.10	$5
$0.25	$10

Expected Value Hour

$.01	$1,000
$1	$5,000
$5	$10,000
$10	$25,000
$25	$50,000
$50	$75,000
$75	$100,000
$100	$200,000
$200	$300,000
$300	$400,000
$400	$500,000
$500	$750,000
$750	$1,000,000

Expected Value Hour

$.01
$1
$5
$10
$25
$50
$75
$100
$200
$300
$400
$500
$750

A "fair deal":

Multiply each outcome by its probability…

Total: $410,210

Fair deal: ~$102,500

$1,000
$5,000
$10,000
$25,000
$50,000
$75,000
$100,000
$200,000
$300,000
$400,000
$500,000
$750,000
$1,000,000

Expected Value Hour

$.01	The "bank offer":	$1,000
$1		$5,000
$5		***$10,000***
$10	Guarantee, almost always less than fair value	$25,000
$25		$50,000
$50		$75,000
$75		$100,000
$100		$200,000
$200	Fair deal: ~$102,500	$300,000
$300		***$400,000***
$400	Offer: $82,000	$500,000
$500		$750,000
$750	*Deal* or *No Deal*?	$1,000,000

Expected Value Hour

$.01
$1
$5
$10
$25
$50
$75
$100
$200
$300
$400
$500
$750

What's the expected value of the initial board?

How does it compare to the first offer?

How does it compare to how much money players actually win?

$1,000
$5,000
$10,000
$25,000
$50,000
$75,000
$100,000
$200,000
$300,000
$400,000
$500,000
$750,000
$1,000,000

Expected Value Hour

$.01
$1
$5
$10
$25
$50
$75
$100
$200
$300
$400
$500
$750

Initial board…

Fair deal: $131,477

First offer:

~$8,000-$20,000

The first offers are *terrible!* Why?

$1,000
$5,000
$10,000
$25,000
$50,000
$75,000
$100,000
$200,000
$300,000
$400,000
$500,000
$750,000
$1,000,000

Expected Value Hour

$.01
$1
$5
$10
$25
$50
$75
$100
$200
$300
$400
$500
$750

Actual average winnings per player:

$122,500

Initial board's expected value:

$131,477

(Close! Why the difference?)

$1,000
$5,000
$10,000
$25,000
$50,000
$75,000
$100,000
$200,000
$300,000
$400,000
$500,000
$750,000
$1,000,000

Math in Game Shows

Game shows are filled with math problems…

- Contestants
  - How do I play best?
  - How much risk should I take?
- Producers
  - How do I build a fun game to watch?
  - How will contestants behave?
  - How much money are we giving out?

Personal Encounters

February 2000: *Millionaire* (episode #49)

(for $1000: How many degrees in a right angle?)

Personal Encounters

February 2000: *Millionaire* (episode #49)

(Got the next question wrong. Yes, that was my real hair.)

Personal Encounters

April 2004: *The Price Is Right*

(Double overbid on the showcase! Bummer.)

Personal Encounters

May 2007: *National Bingo Night*

(We worked on this show a lot longer than it lasted.)

Personal Encounters

August 2012: *Oh Sit!*

(Wipeout + musical chairs + Jamie Kennedy = ???)

Personal Encounters

June 2014: *Sing Your Face Off*

(Even *this* needed a mathematical advisor.)

Let's Play!

We're picking one contestant for this game.

We'll also need *all sixteen* volunteers from the audience to help us with the game.

John Steiner, *come on down!*

Sponsored by… CME Project

- NSF-funded curriculum from EDC / Pearson
- ~ 100,000 students nationally
- Common Core State Standards: 100% alignment

Now available in new integrated flavor!

The widespread utility and effectiveness of mathematics come not just from mastering specific skills, topics, and techniques, but more importantly, from developing the ways of thinking —the habits of mind—used to create the results.

CME Project Overview

By focusing on *habits of mind*…

- Coherent curriculum, fewer chapters
- CME was 95% aligned to CCSSM content standards *at the time the standards were first published*
- Especially strong alignment with MPs
- CCSSM used CME Project's language in writing MPs!

cmeproject.edc.org

(we also do house calls… but now, back to the show)

The Price Is Right

- Now in its 43rd year
- Lots of good math problems!
- Huge sample size of repeated play

`tpirstats.com`

1/2 Off

There are 16 boxes, uh, *people*.

One has a big prize.

You'll have 3 chances to eliminate half the people.

Choice #1

Which of these is *half off* the actual price?

Chowda

$6.00

Lobsta Roll

$9.50

Choice #1

Which of these is *half off* the actual price?

Chowda

$6.00

Lobsta Roll

$19.00

Choice #2

Which of these is *half off* the actual price?

Slurpee

$1.50

Bottled Water

$2.00

Choice #2

Which of these is *half off* the actual price?

Slurpee

$1.50

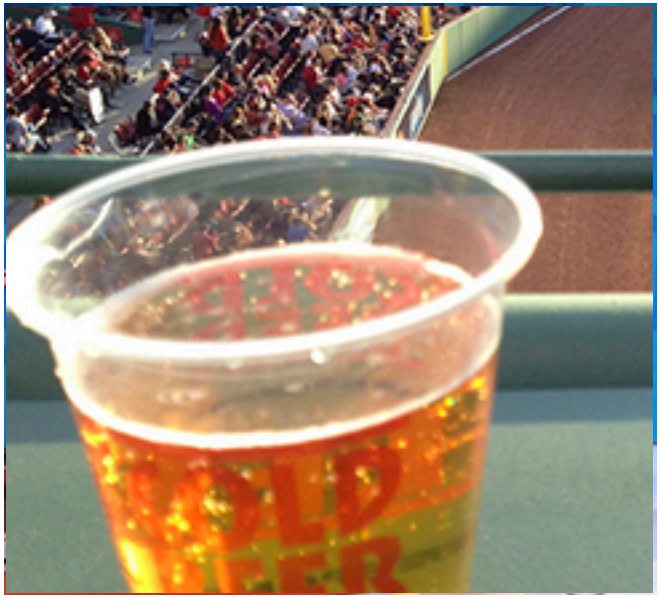
Bottled Water

$4.00

Choice #3

Which of these is *half off* the actual price?

Small Beah



$4.00

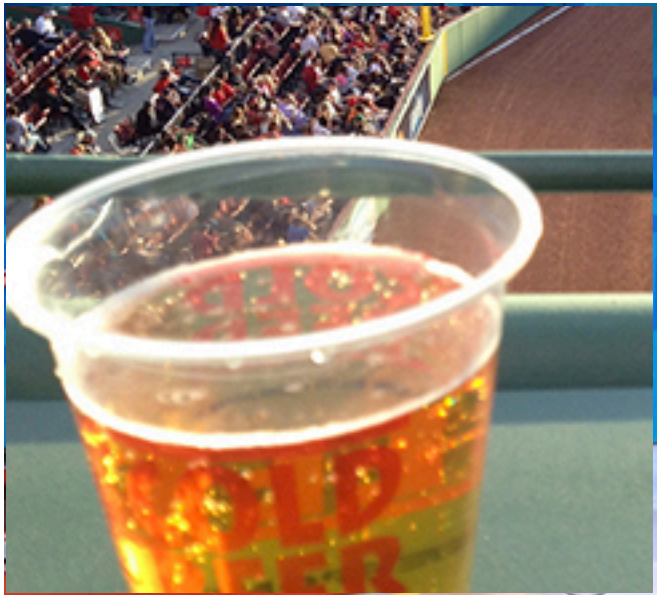
Fenway Frank

$5.25

Choice #3

Which of these is *half off* the actual price?

Small Beah



$8.00

Fenway Frank

$5.25

Prize Sponsored by MTBoS

The Math Twitter Blog-o-Sphere is an awesome place to hang out virtually and talk math.

Stop by!

841

Nix the Tricks by Tina Cardone is awesome!

The Producers' Questions

If we keep offering this game repeatedly,
how much will we have to pay for it?

How likely is a win *if the player guesses*…

0 right? 1 right? 2 right? 3 right?

(and the most important question…)

The Producers' Questions

If we keep offering this game repeatedly,
how much will we have to pay for it?

How likely is a win *if the player guesses*…

0 right? 1 right? 2 right? 3 right?

Is this game fun to watch??

Analysis: 1/2 Off

The number of correct answers determines the probability of winning.

# Correct	P(win)
0	1/16
1	1/8
2	1/4
3	1/2

Hooray. Now what?

Analysis: 1/2 Off

The normal 1/2 Off prize is $10,000, plus a $1,000 bonus for getting all 3 right.

#	Avg. Winnings
0	1/16 * 10000 = $625
1	1/8 * 10000 = $1250
2	1/4 * 10000 = $2500
3	1/2 * 10000 + 1000 = $6000

Analysis: 1/2 Off

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#	Avg. Winnings
0	1/16 * 10000 = $625
1	1/8 * 10000 = $1250
2	1/4 * 10000 = $2500
3	1/2 * 10000 + 1000 = $6000

Can we just average these four numbers?

Historical Data

1/2 Off has been played 187 times since 2000, fully detailed on tpirstats.com.

2000-2014

0 correct: 5.9% (11 times)

1 correct: 24.1% (45 times)

2 correct: 41.2% (77 times)

3 correct: 28.9% (54 times)

Using Historical Data

Use these percentages for the weighted average.

#	Contribution to EV	Product
0	$625 * 5.9%	$37
1	$1250 * 24.1%	$301
2	$2500 * 41.2%	$1030
3	$6000 * 28.9%	$1734
	total:	*$3102*

Using Historical Data

Use these percentages for the weighted average.

#	Contribution to EV	Product
0	$625 * 5.9%	$37
1	$1250 * 24.1%	$301
2	$2500 * 41.2%	$1030
3	$6000 * 28.9%	$1734
	total:	*$3102*

Actual win rate:

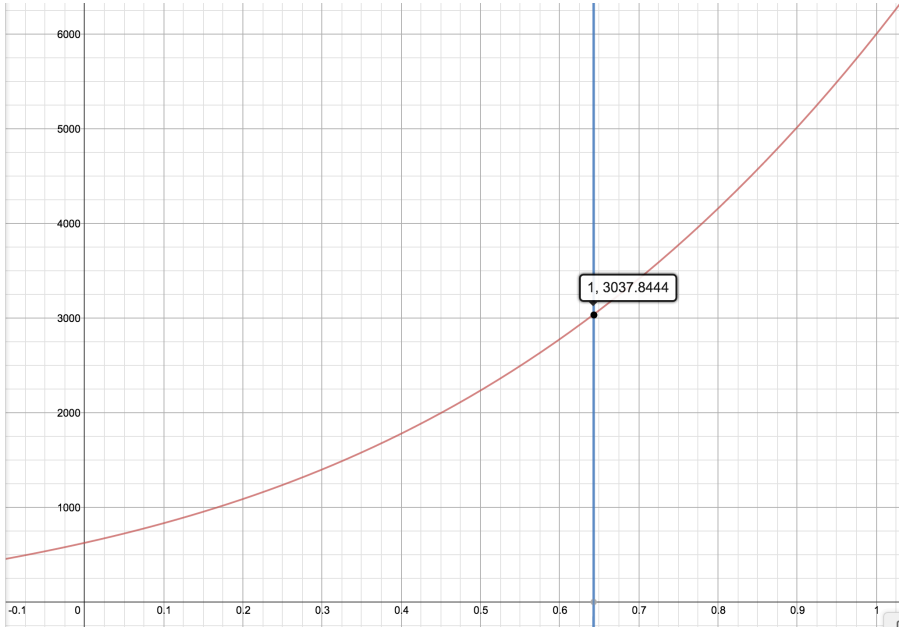
31.02%

(wow!)

Using Algebra

If the change of getting a choice right is *p*…

#	Contribution to EV
0	$625*(1-p)^3$
1	$1250*3p(1-p)^2$
2	$2500*3p^2(1-p)$
3	$6000*p^3$



Players choose with $p \approx .643$

$f(.643) \approx \$3038$

What's In The Box??

After 187 plays, where is the money?

9	15	7	21
7	16	8	16
9	9	8	11
3	21	12	15

Sponsored by… Marshmallow Fluff

Marshmallow Fluff:

The second best thing to ever come out of Lynn, MA

Try a Fluffernutter! No, seriously, they're awesome.

Let’s Play!

Sara Cafarelli… come on down!

Master Key

There are 5 keys.

One key unlocks each of three prizes.

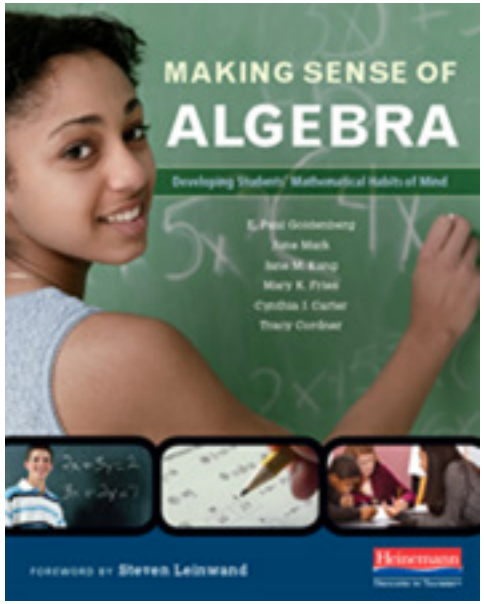
One key is the *Master Key* and opens it all.

One key is a dud.

You'll have 2 chances to earn a key.

Prizes Sponsored by Heinemann

Transition To Algebra raises the competence and confidence of first-year algebra students



Stop by!

834-835

Making Sense Of Algebra

just published!

Choice #1

How many member states are there in the Smarter Balanced consortium?

121

12 or 21?

Choice #1

How many member states are there in the Smarter Balanced consortium?

121

Choice #2

How many total chapters are there in the four *CME Project* books?

432

43 or 32?

Choice #2

How many total chapters are there in the four *CME Project* books?

4**32**

Pick Keys

(Hopefully, you won at least one key…)

Analysis: Master Key

Let's say we only care about the big prize.

# keys	P(win big prize)
0	0
1	2/5 = 40%
2	Hm…

Analysis: Master Key

Let's say we only care about the big prize.

# keys	P(win big prize)
0	0
1	2/5 = 40%
2	Hm…

What about finding P(lose if 2 keys)?

Analysis: Master Key

Let's say we only care about the big prize.

# keys	P(win big prize)
0	0
1	2/5 = 40%
2	7/10 = 70%

*P(lose if 2 keys) = 3/5 * 2/4*

Historical Data

Master Key has been played 111 times.

# keys	P(win big)	P(actual)
0	0	
1	40%	
2	70%	

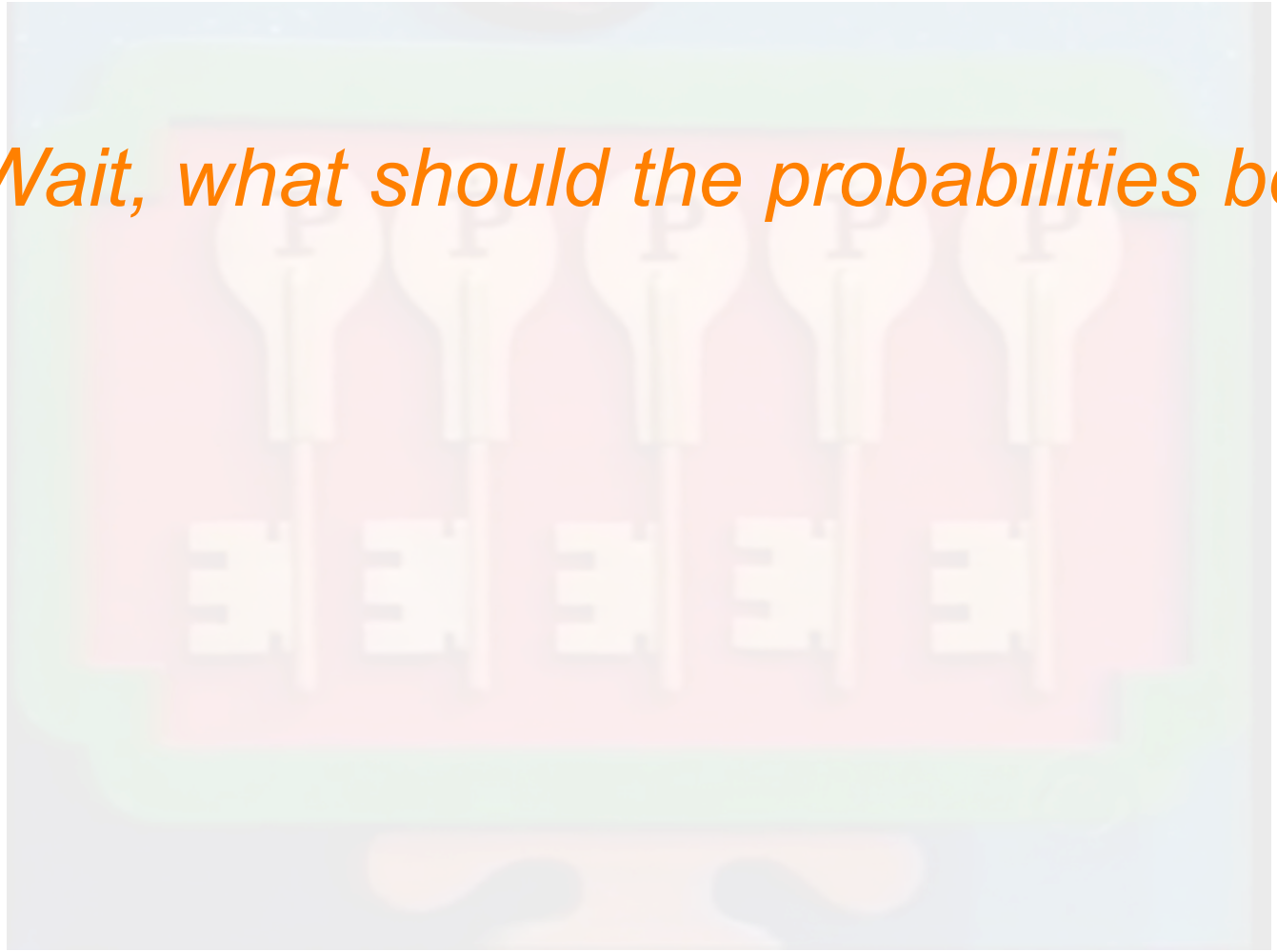
Historical Data

Master Key has been played 111 times.

# keys	P(win big)	P(actual)
0	0	0
1	40%	*30.5%*
2	70%	67.7%

Where are the Keys?

Here's the probability for each key to win big.



Wait, what should the probabilities be?

Where are the Keys?

Here's the probability for each key to win big.

If everything was "equal"…

40%	40%	40%	40%	40%

Where are the Keys?

Here's the probability for each key to win big.

The actual probabilities…

92%	17%	21%	23%	70%

(Holy...)

The Choices, Too…

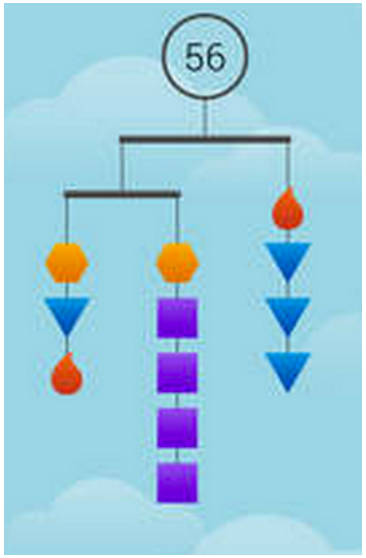
We asked for "left" or "right".

50/50? Hardly.

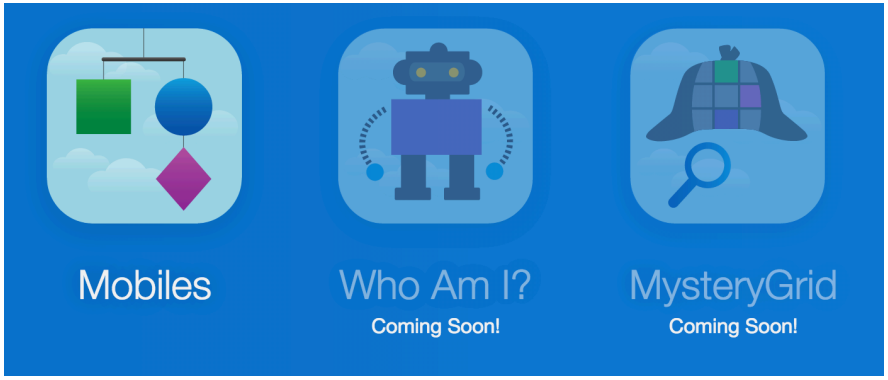
In Master Key, 78.2% of the correct choices are to the right.

Sponsored by… SolveMe!

Hundreds of puzzles to play … or make your own!



It's fun and teaches equation solving! Oh, and it's *FREE* for iPad.



solveme.edc.org

Classroom Interlude

In my teaching, I found some game shows worked better than others. Games are great test review! Good as openers / wrap-ups.

Good	*Bad*
Press Your Luck	Jeopardy! *(yes, bad)*
Card Sharks	Deal or No Deal
Millionaire	Wheel of Fortune
High Rollers	Studs

Classroom Interlude

Here are a few potential projects to try.

- Make a game with P(win) ≈ 1/3.
- What are good wagers in Final Jeopardy?
- What other Price Is Right games could be played better through strategy? (Slate)
- What's the probability of winning $1 million on Wheel of Fortune?

Let's Play!

(It's the last game, let's make it a good one…)

Hannah… come on down!

You're going to play…
PLINKO!

With a chance to win…

You're going to play…

PLINKO!

With a chance to win…

Prize Sponsored by… EDC's Mathematical Practice Institute

- EDC's professional development program
- Curriculum-neutral, focused on Standards for Mathematical Practice
- *PD on your schedule: one day, multi-day*
- *Online webinars available*

Visit the MPI website: *mpi.edc.org*

That website again is: *mpi.edc.org*

A Plinko Dilemma

The Price Is Right
@PriceIsRight

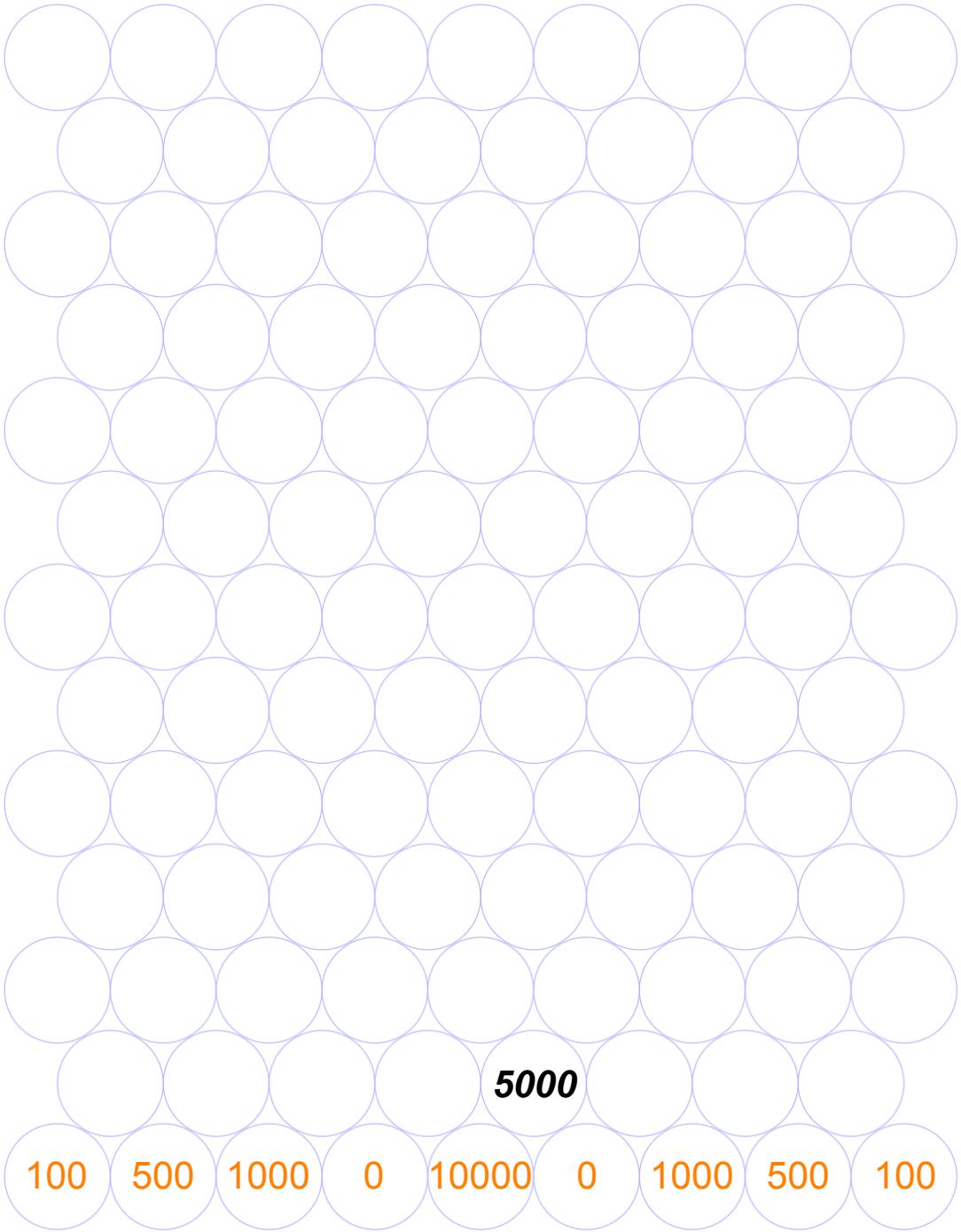
Following

Should we make the center number on #Plinko board $25,000 or make the ends worth $10,000 each? #UDecide #PriceIsRight

Backtracking Plinko

How much is this Plinko chip worth right now?

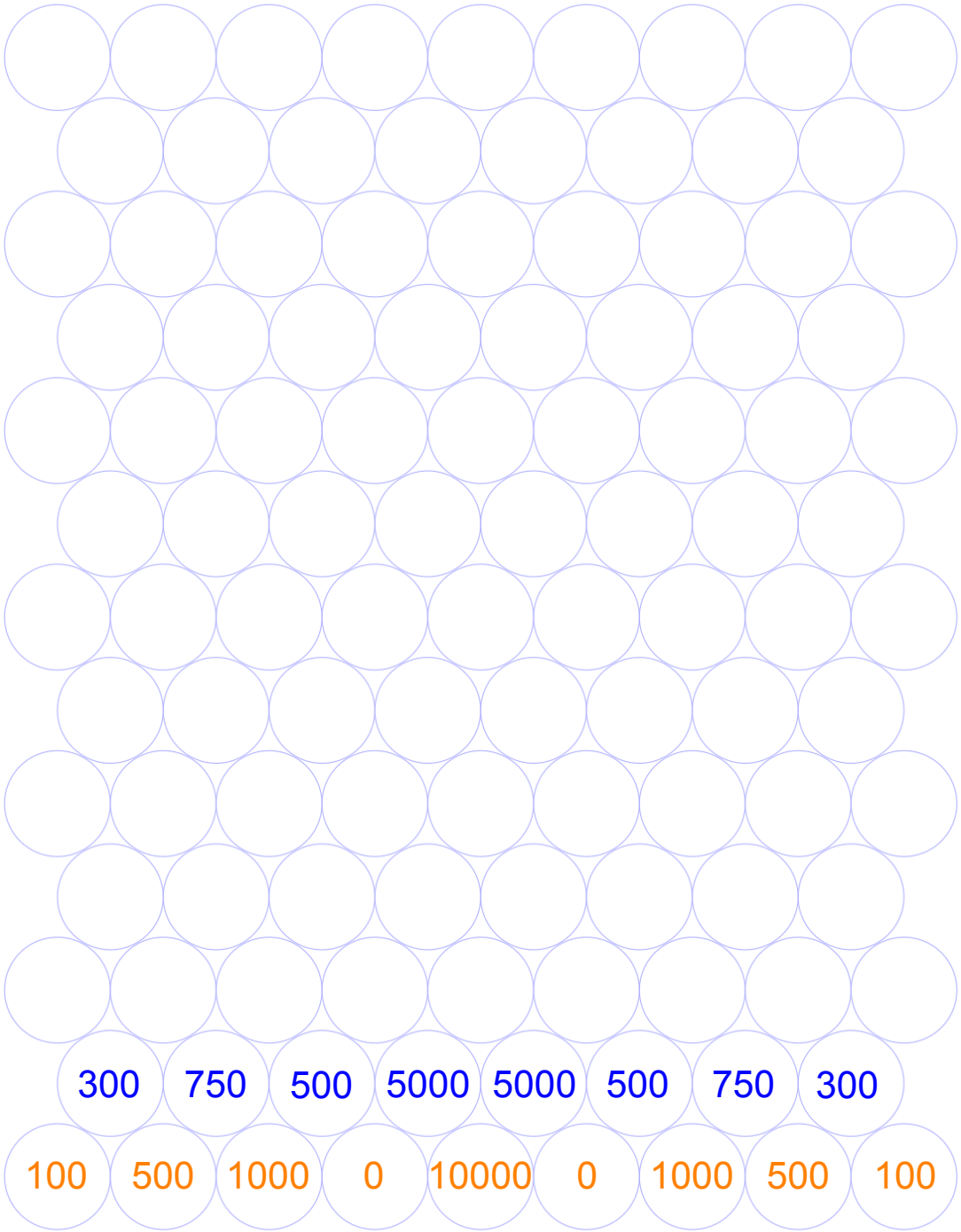
Backtracking Plinko



Each entry is the value of a chip at that spot.

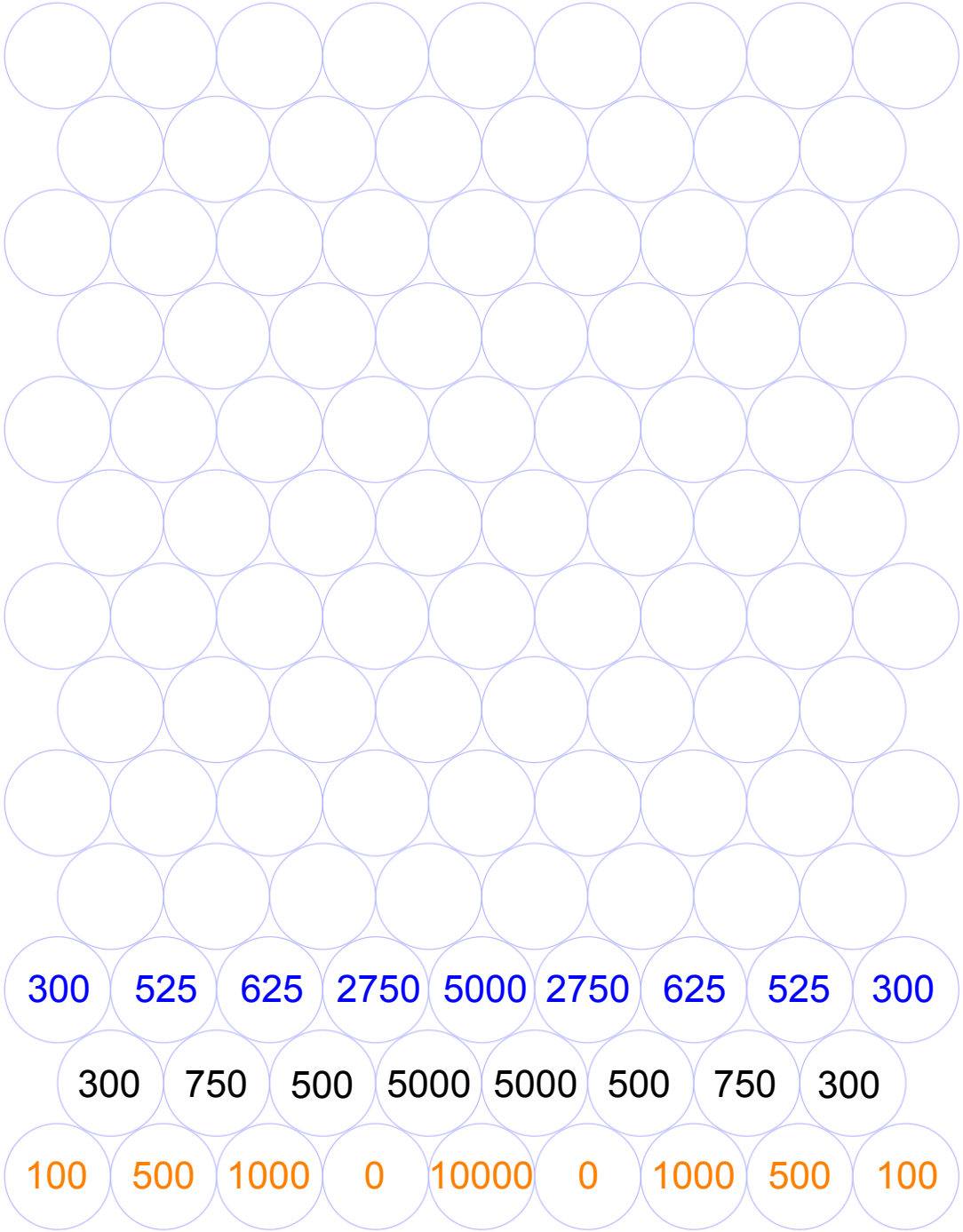
We know the last row…

Backtracking Plinko



Work from the bottom up… each number is the mean of the two below it!

Backtracking Plinko



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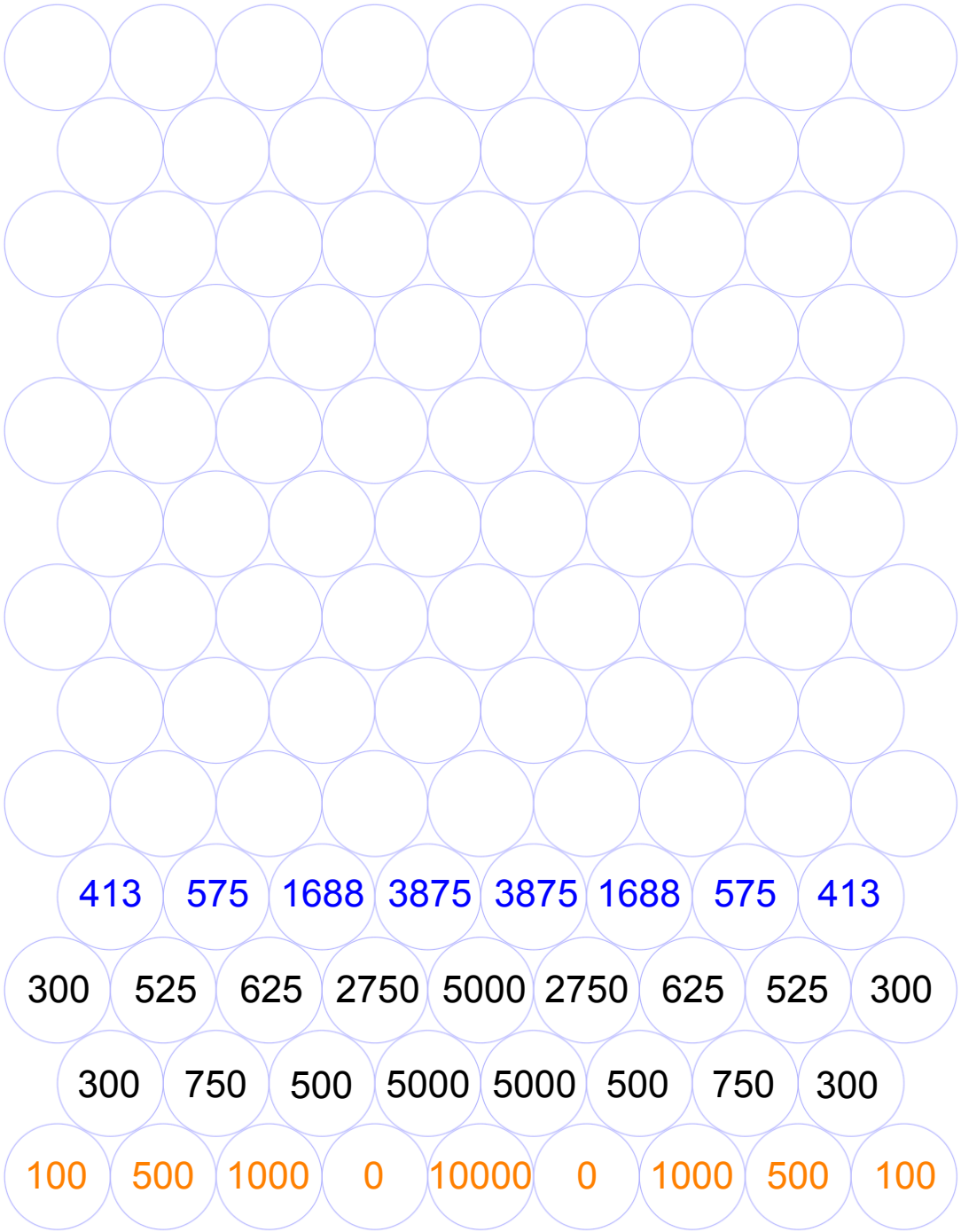

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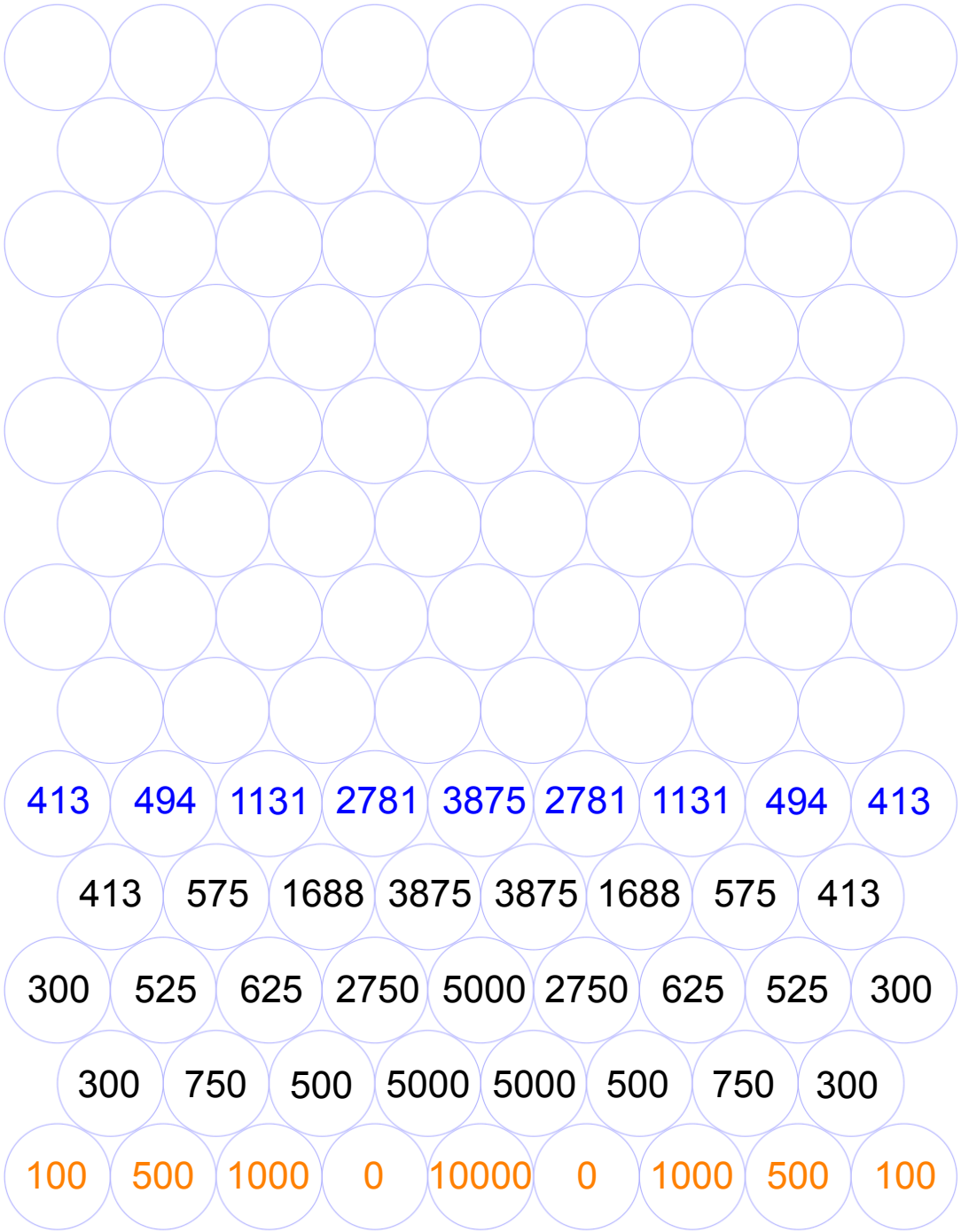
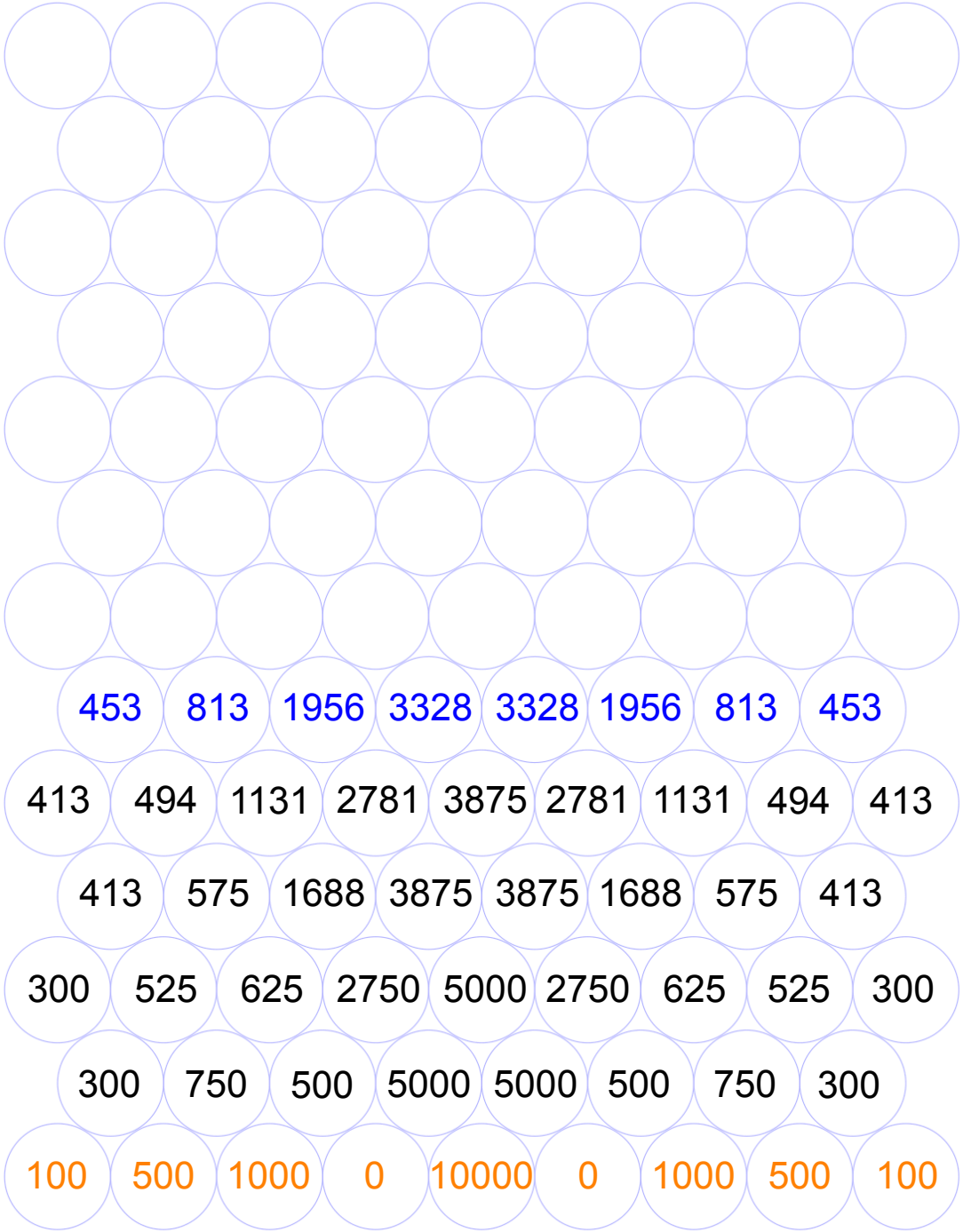
Backtracking Plinko



Work from the bottom up… each number is the mean of the two below it!

Backtracking Plinko



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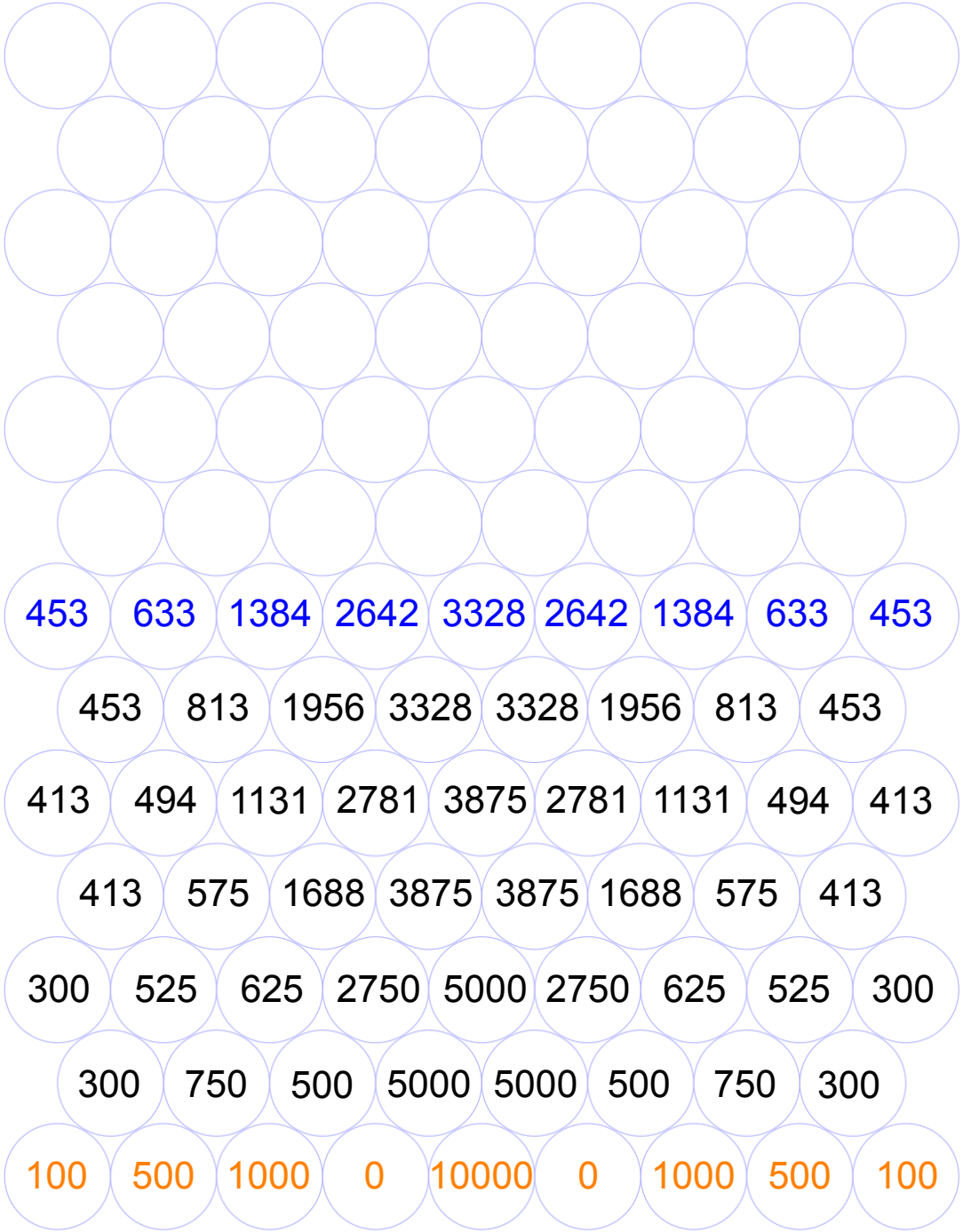
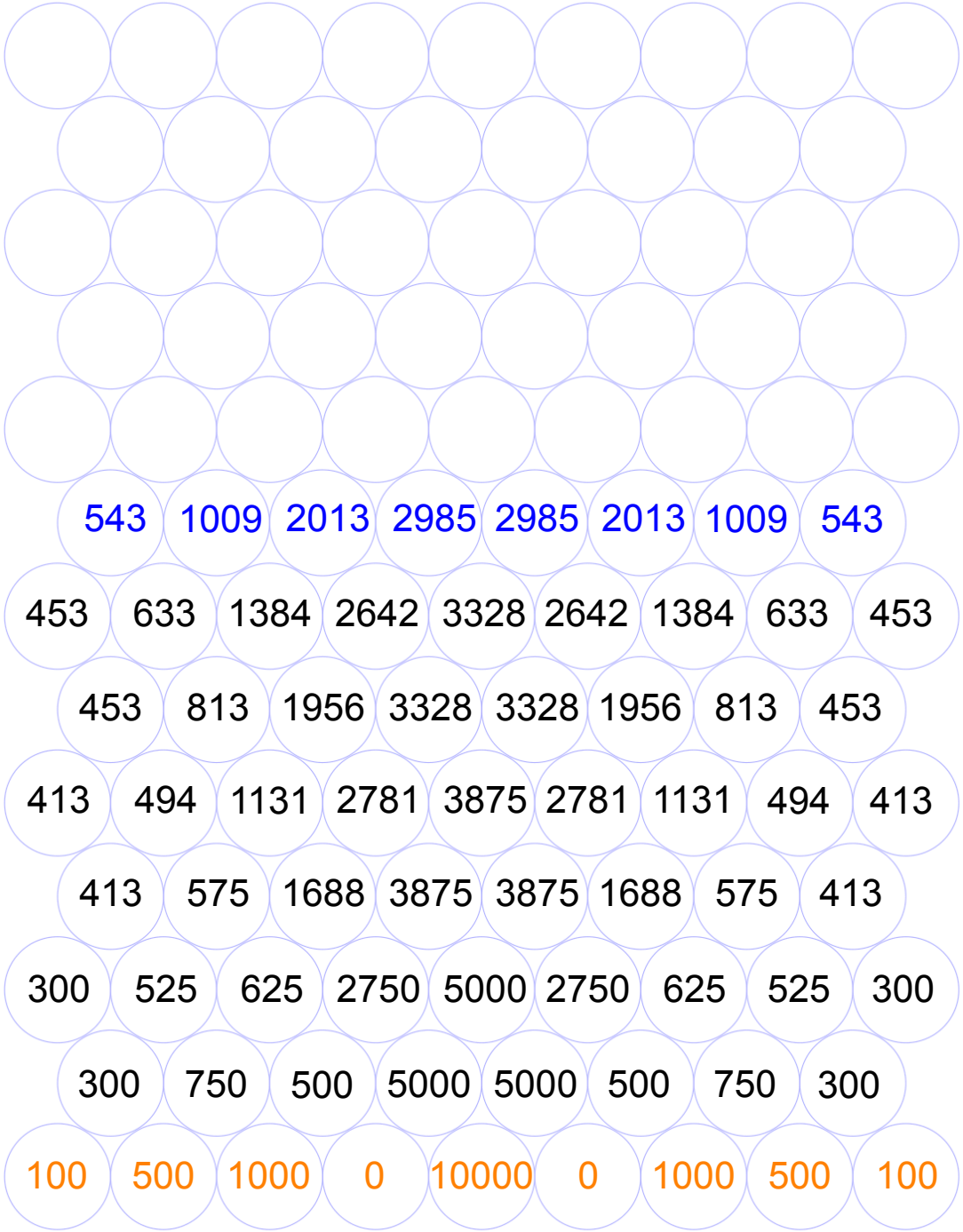


In the long run, it all evens out.

But this isn't a long run…

Backtracking Plinko



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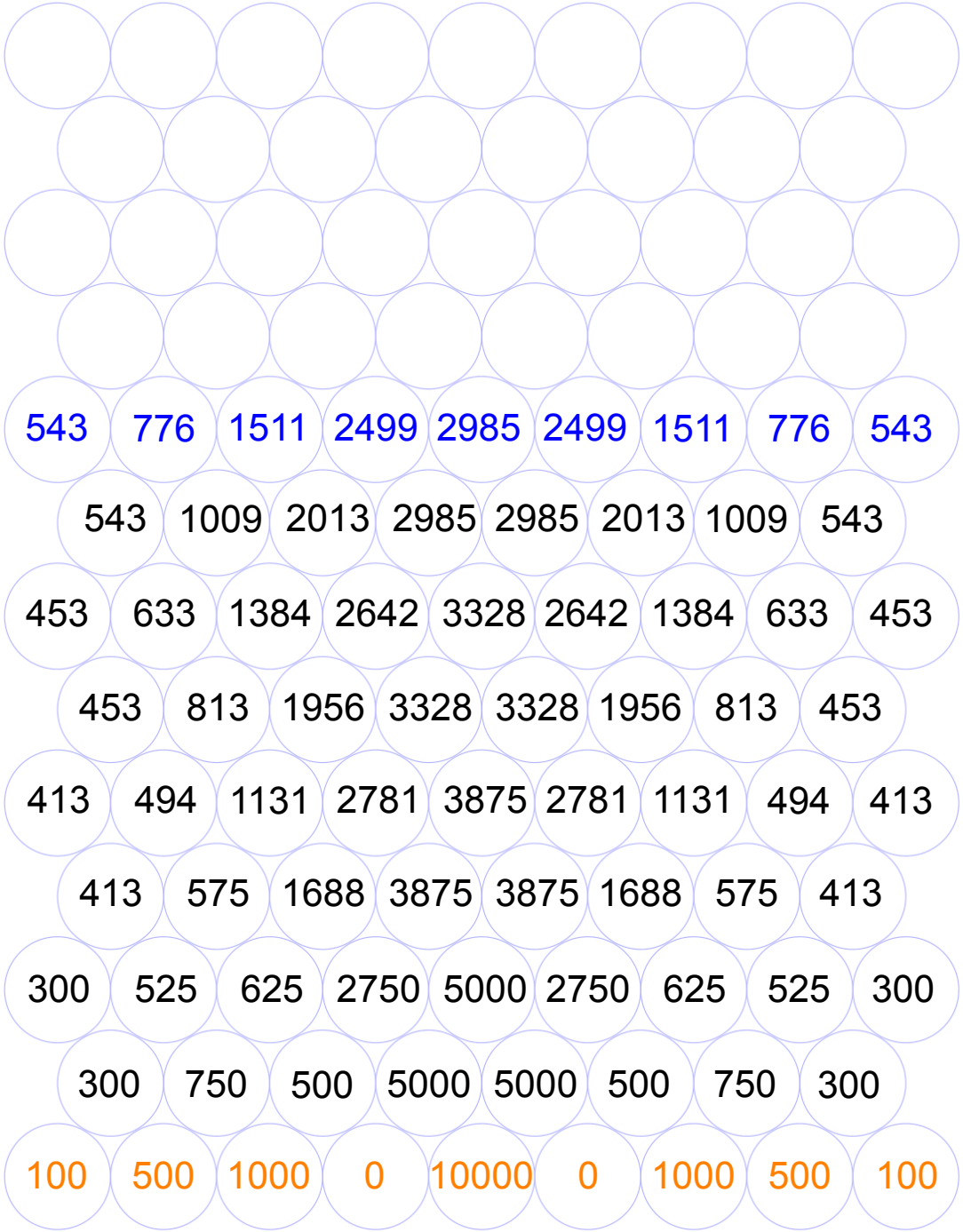

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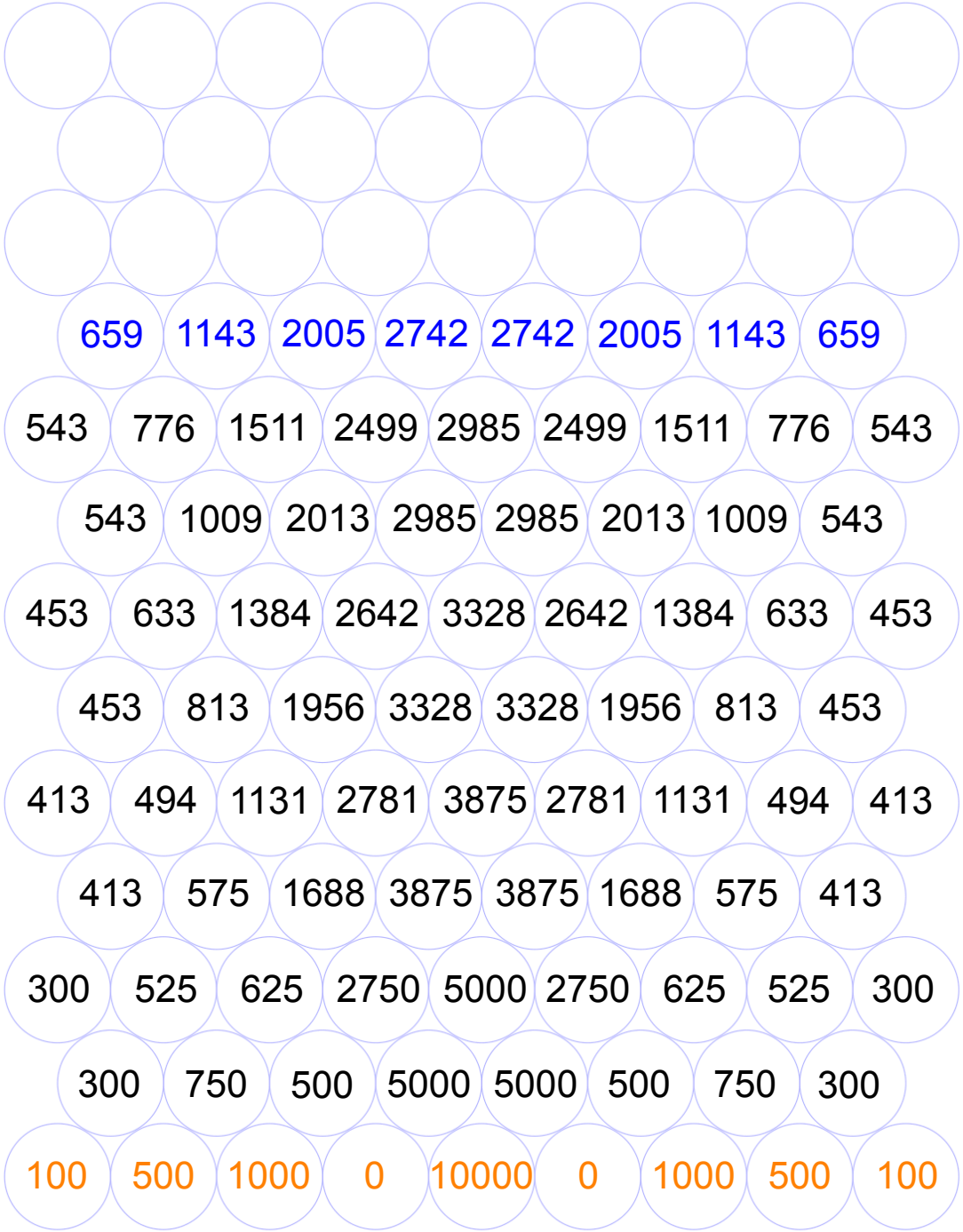

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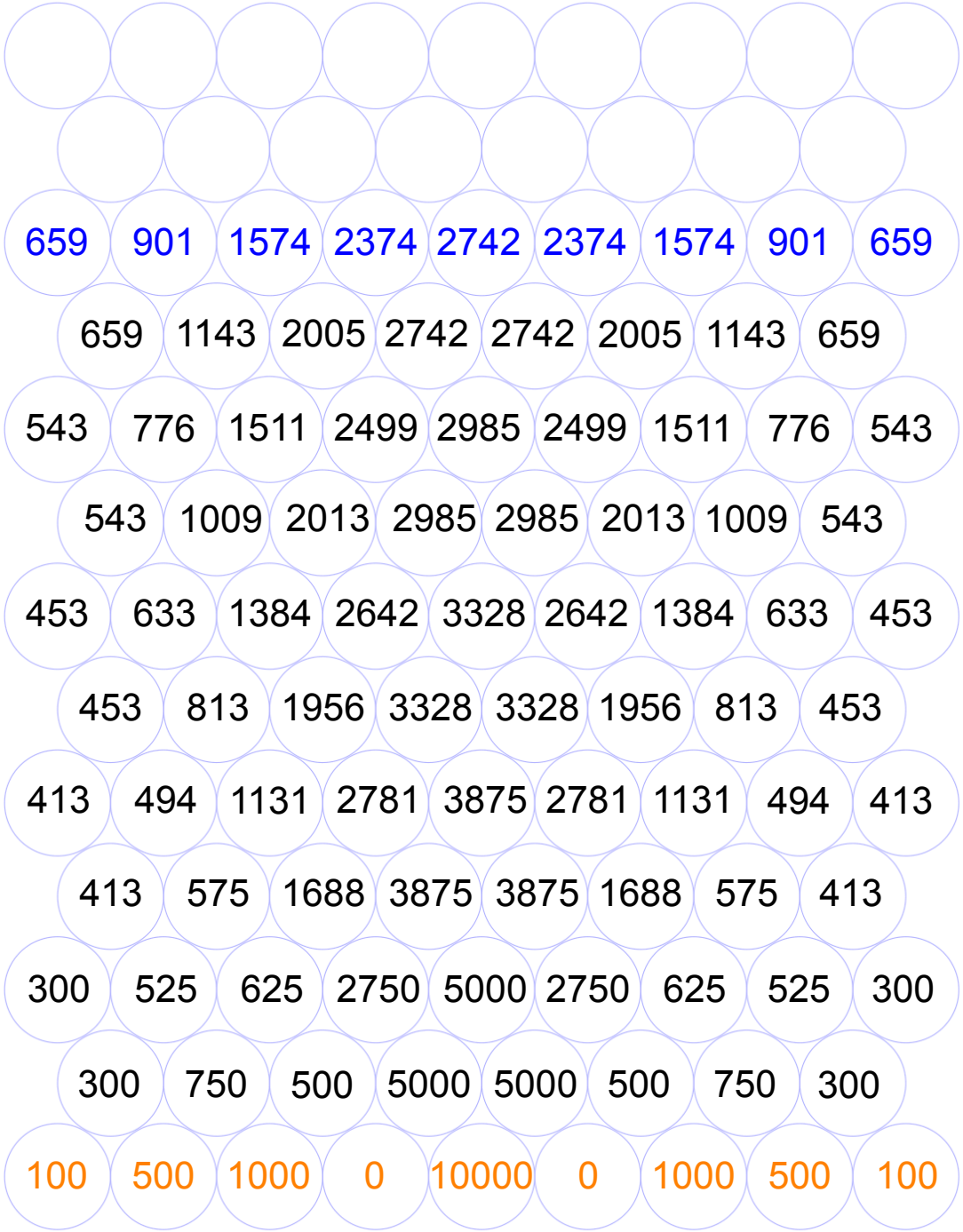

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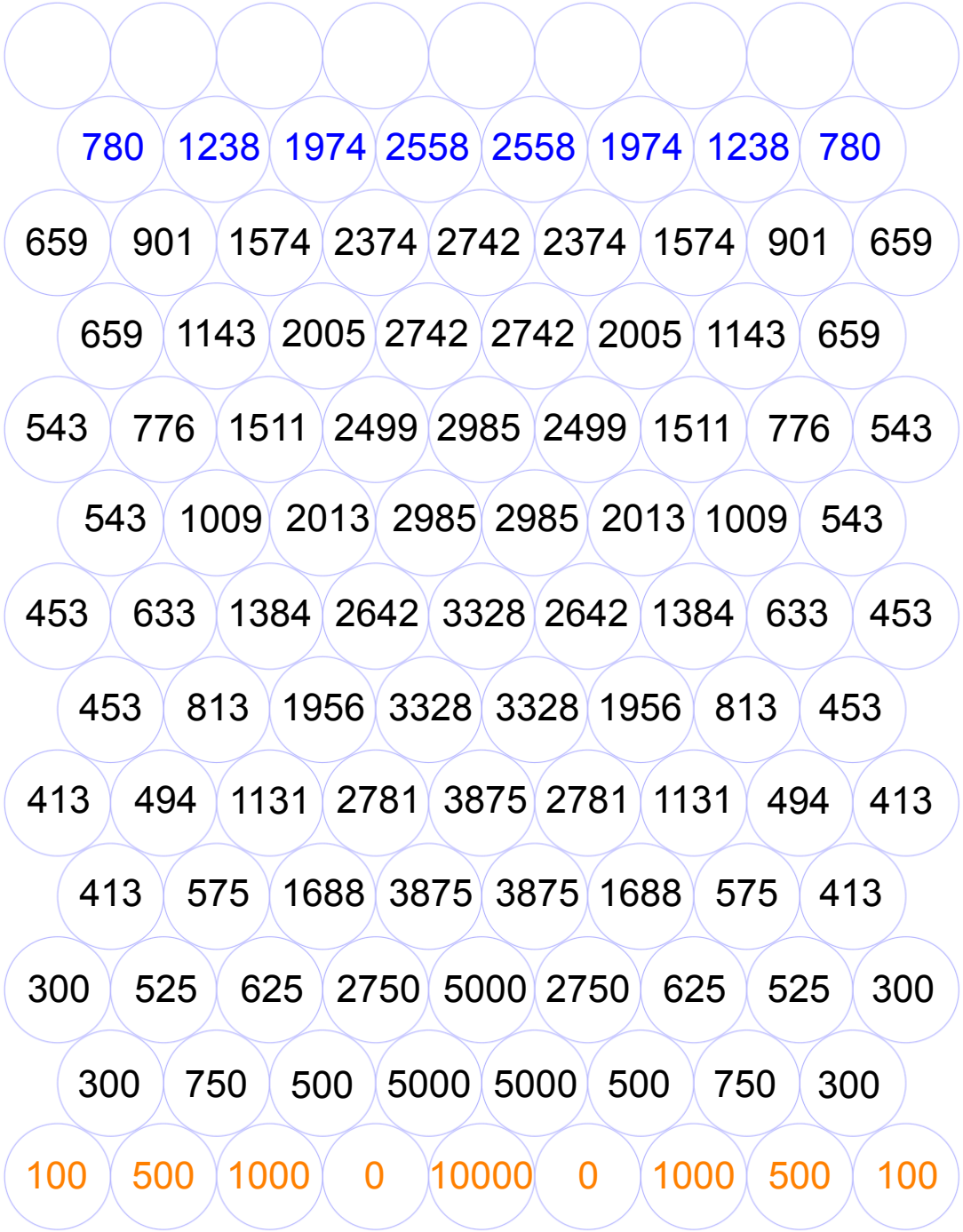
Backtracking Plinko



At the top we find the expected value for dropping a chip from each slot!

Backtracking Plinko

780	1009	1606	2266	2558	2266	1606	1009	780
780	1238	1974	2558	2558	1974	1238	780	
659	901	1574	2374	2742	2374	1574	901	659
659	1143	2005	2742	2742	2005	1143	659	
543	776	1511	2499	2985	2499	1511	776	543
543	1009	2013	2985	2985	2013	1009	543	
453	633	1384	2642	3328	2642	1384	633	453
453	813	1956	3328	3328	1956	813	453	
413	494	1131	2781	3875	2781	1131	494	413
413	575	1688	3875	3875	1688	575	413	
300	525	625	2750	5000	2750	625	525	300
300	750	500	5000	5000	500	750	300	
100	500	1000	0	10000	0	1000	500	100

At the top we find the expected value for dropping a chip from each slot!

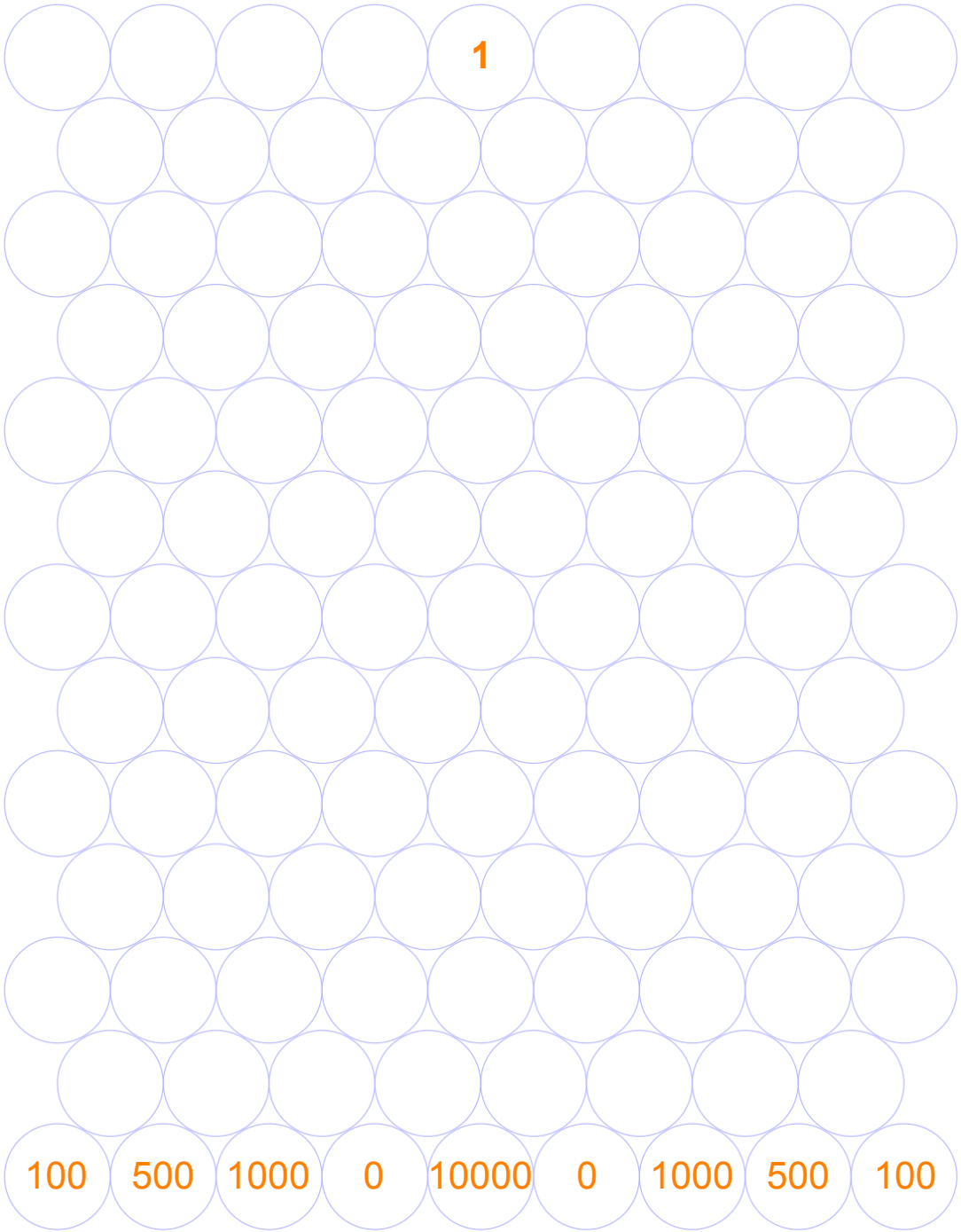
Plinko Advice

Where you drop Plinko chips matters a lot!

Drop Above	Chip EV
$10,000	$2,558
$0	$2,266
$1,000	$1,606
$500	$1,009
$100	$780

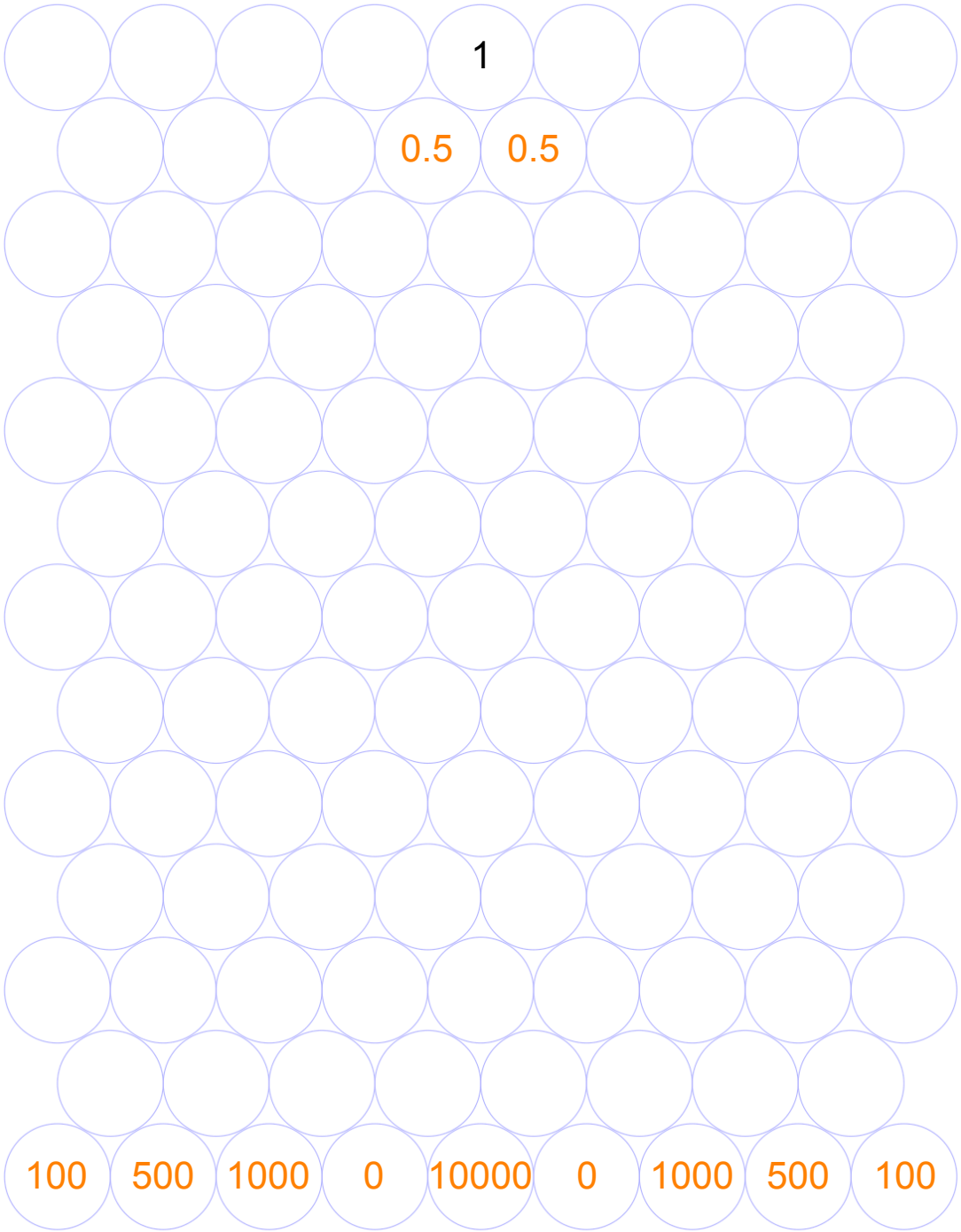
So… about that dilemma...

From Top To Bottom



Now each entry is the probability of entering that spot…

From Top To Bottom



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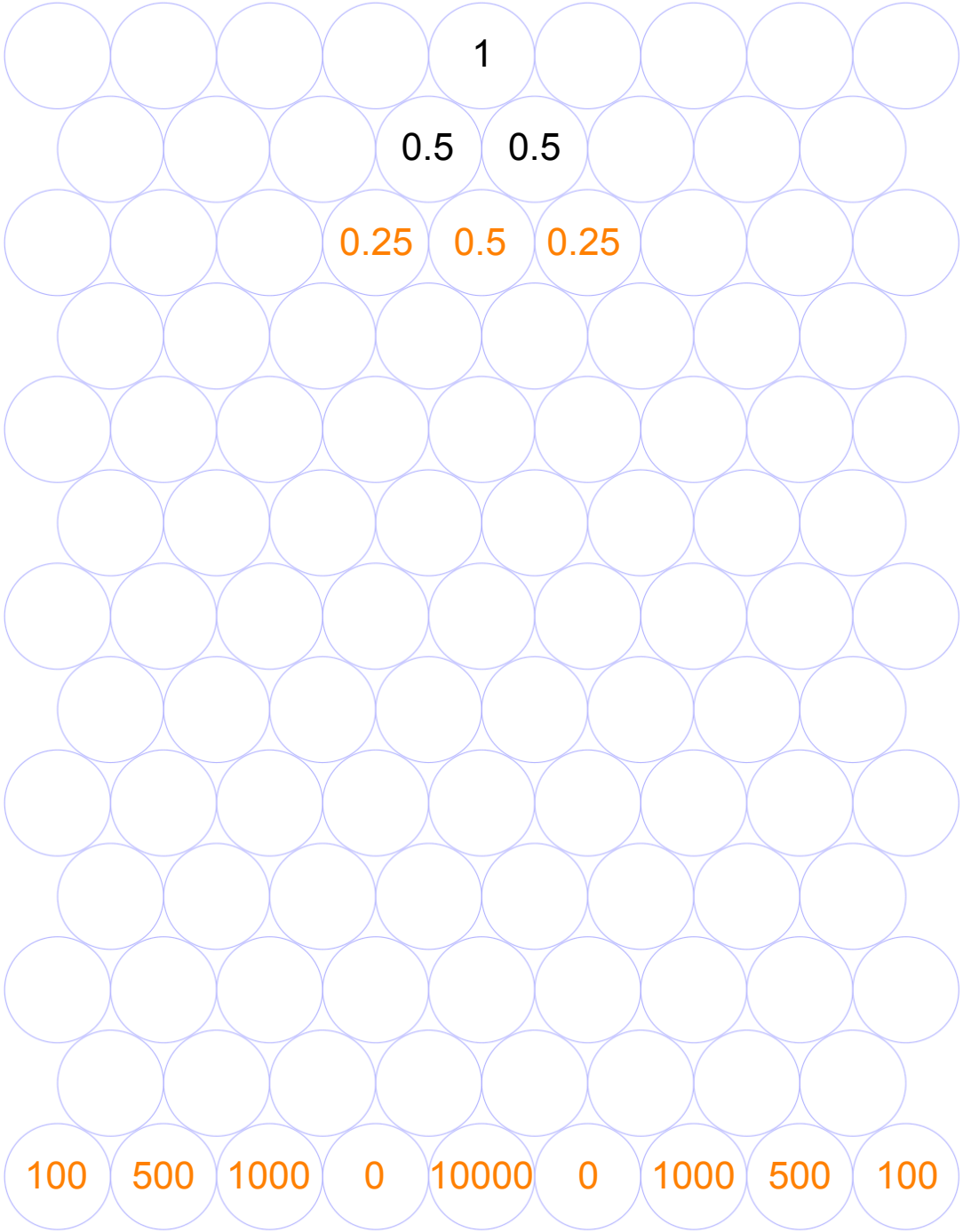
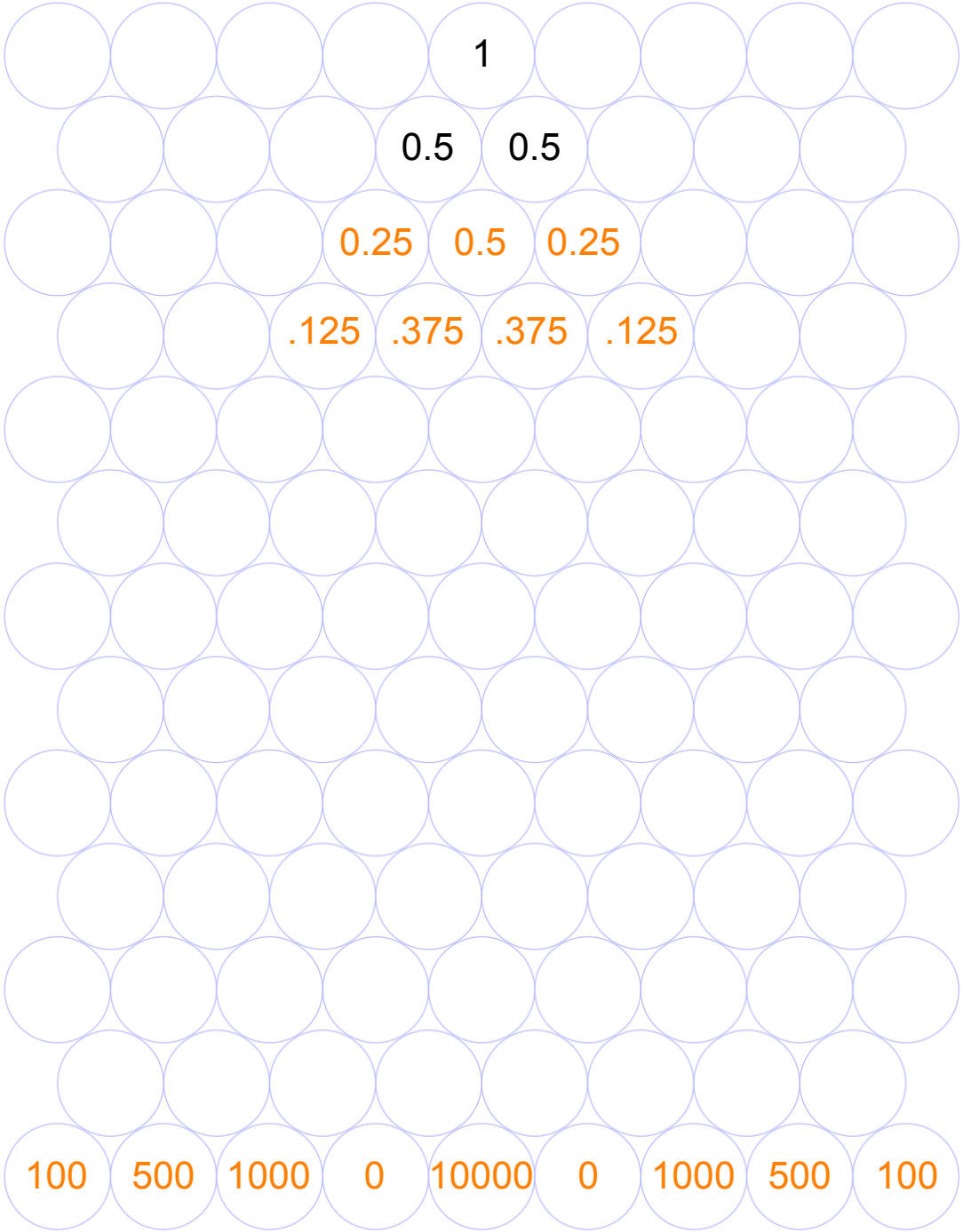
From Top To Bottom



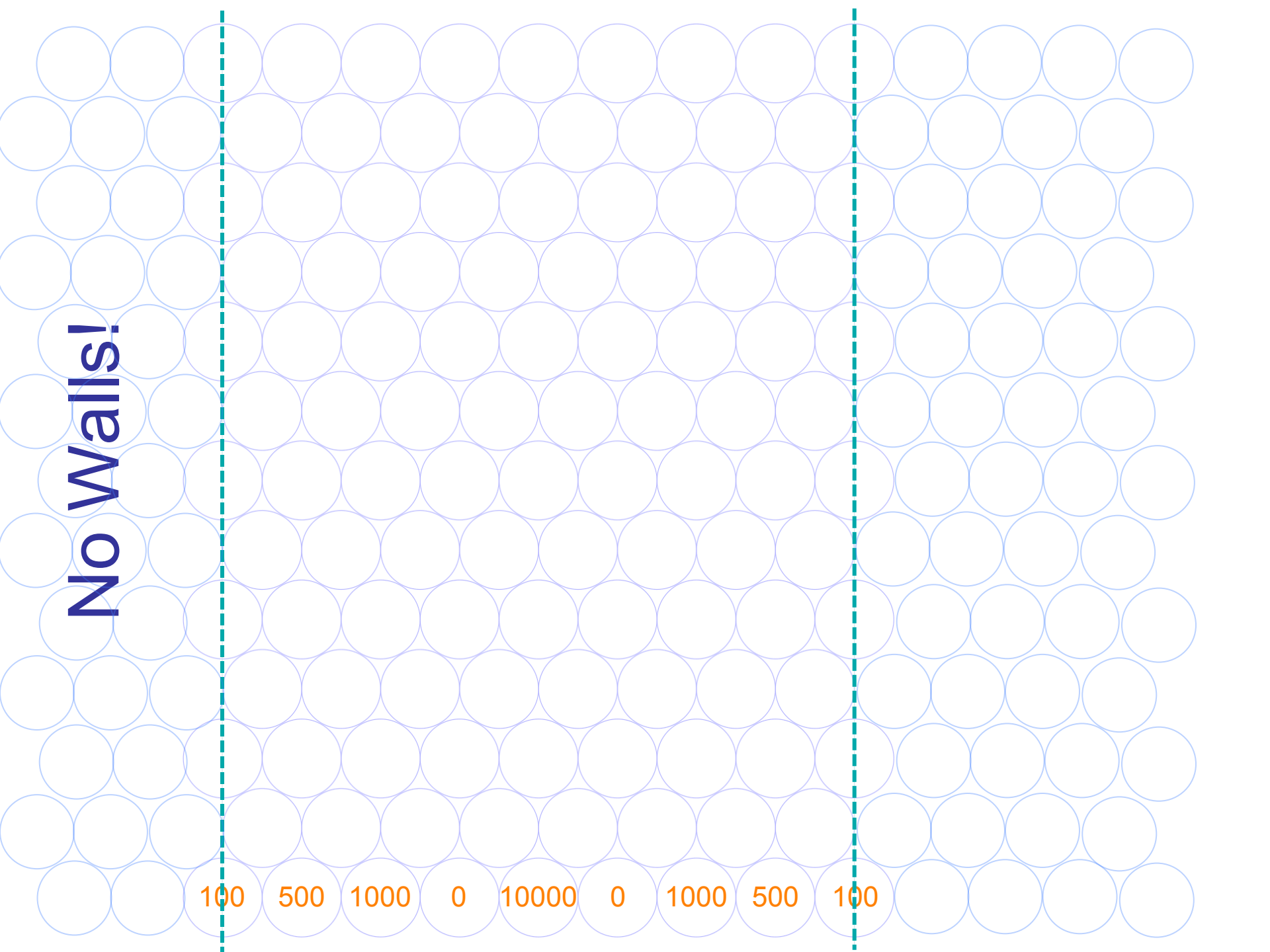
… and you still use the mean to calculate the next row!

From Top To Bottom



Cool, but what about the walls?

Sure would be nice if there were no walls…


No Walls!
100
500
1000
0
10000
0
1000
500
100

More to Explore

Many related topics are asked about in *CME Project*, and in the Park City Math Institute materials at

www.mathforum.org/pcmi/hstp/sum2013/morning

- How can spinners or dice be represented by polynomials?
- How can you use Pascal's Triangle on Plinko? Does "No Walls" help somehow?
- What's the best possible total in an episode of Jeopardy?

eCMI

- Inspired by the *Park City Mathematics Institute*
- eCMI is live, online, concurrent PD at multiple sites
- Currently in pilot testing
- Sign up for free!

Sign up at the eCMI website:

ecmi.edc.org

(Enough commercials, already.)

Thanks and good luck!
Any questions?

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